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Estimation of Oil and Gas Reserves in Place using Production Decline Trend Analysis

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Abstract- Mathematical models equations were successfully derived for studying reservoirs fluids depletion from the peak value to an economic value called abandonment. Estimation of Oil and/or Gas Cumulative Production and Initially in Place was done Using Production Decline Rate Trend Analysis. Fields past production data (called regional data) were used to generate standard curves. These curves were empirically used in generating the evaluation model equations. The evaluation model equations were used to project future hydrocarbons production rates in a given time. The projected rates values (called generic or projected data) were also plotted against time, which generated curves that were equally used to obtain the prediction model equations for the estimation of the cumulative hydrocarbons production and the hydrocarbons initially in place. The estimated cumulative hydrocarbons productions were comparable with the respective tank values. The percentage accuracy for gas fields ranged from 99.86% and above, while the percentage accuracy for oil ranged from 98.64% to 99.98%. The models developed for decline rates trends using field data were the basic tools which showed high percentage of accuracy. The advantage of using projectiles and parabolic methods in model development is that such models would be very flexible. The models could be applied with high accuracy right from the initial reservoir stage, through the transient stage and transition stage to the decline rate stage.

Keywords- *Estimation, Production, Trend Analysis, Oil and Gas*

I. INTRODUCTION

Oil and gas projected production and initially in place estimation is a serious business that should not be taken for granted. This problem can result in ineffective recovery factor, adequate economic evaluation of wells performances and early abandonment of wells due to wrong production forecast. The cumulative production forecast and the accurate estimation of hydrocarbons initially in place is a key to fluid recovery optimization. To this effect serious ideas must be cultivated in order to improve the accuracy in hydrocarbons recovery estimation and initially in place. Among the factors which influence hydrocarbons production values forecast are the fluids production decline constant (b) and the order/trend of decline rate. The fluid production decline constant depends on two phenomena, the energy controlling the reservoir and the location of producing wells on the phase envelope. If both the external and internal energies of a reservoir drainage or depletion are laminar, the production rate decline could be constant or proportional to the time elapsed. If it is turbulent, the rate decline would be non-uniform or unsteady. In this case the production rate trend would change with the time of production. The disadvantage when a production rate decline changes in a given time is incorrect cumulative hydrocarbons production forecast, especially in an insufficient production data. The hydrocarbons initially in place equally may be wrongly estimated. The economic implication is that the amortization (install mental loan refund) value may be wrongly calculated, thereby making the economy of that contractual agreement of the business unstable and unreliable. The location of the producers above or below the bubble point conditions on the phase envelope has serious influences on the production rate decline trend. If the fluid production wells are located much above the bubble point conditions the production rate would rise to a maximum value, (experiencing a steady flow stage, a transient flow stage, transition flow stage) before it sets into a decline rate trend to an economic minimum value. The advantage in this phenomenon is that the decline rate would be steady or proportionally constant, unless acted otherwise. If the in the other way the production wells are located below the bubble point conditions it means that production rate decline trend may change with the time of production, because initially the internal seemed to contribute on the drive before the external energy pups into pushing it about. The system needs some time for complete combination drive mechanism. The combined influences depend on the rate of external energy invasion effects and its decline trend. When both external and internal energies influence the rate, it is unsteady until complete combination is attained. At the complete combination influencing stage the momentum attains a uniform flow impact. Mathematically:

$$\boldsymbol{M_1V_1 + M_2V_2 = (M_1 + M_2)V = Ft}$$

The LHS of this the equation is the initial impact which depends primarily on the reservoir internal energy, while the RHS is the final impact which depends on the complete combination of the external and internal energies. Later production relies principally on final impact, so the decline rate may attain a different trend due to the effects of the combined impact on the entire reservoir system. So many paper publications on oil and/or gas production rate decline and performances prediction have been done. The theory of the production rate decline is not yet understood. They based most of their performances prediction on the empirical observations of the oil and/or gas production rate decline types. To this extent oil and/or future production or projected values accuracy has prompted a significant level of research for the development of possible solutions.

A. Constant or Exponential Decline Rate

Arps, (1945) used an empirical relationship and analyzed hydrocarbons production decline curves. In his work he defined hydrocarbons production decline rate as a factional change (a) in the flow rate (q) with respect to time (t). His mathematical equations are:

$$\boldsymbol{a = \frac{-{dq}/{dt}}{q_i},\ stb/d\ or\ stb/yr} \tag{1}$$

$$\boldsymbol{N_p = \frac{q_i - q}{a}} \tag{2}$$

AArps, (1956) used his models in the prediction of oil wells production decline rate types. Here Arps pointed out that there are 3-main types of production decline rate power constants (n). Constant or exponential decline rate, n = 0, hyperbolic decline rate, $0 < n < 1.0$ and harmonic decline rate, n = 1.0. He plotted production data against time in a semi-log paper and found out that it gives a straight line graph which could be extrapolated to estimate the oilfield reserves.

$$\boldsymbol{a = -\frac{dq}{q\,dt} = constant} \tag{3}$$

Spivey, et al (1992) gave his as:

$$\boldsymbol{Q_{Dd} = 1 - e^{-t_{Dd}} = 1 - q_{Dd}} \tag{4}$$

B. Hyperbolic and Harmonic Decline Rate

In the hyperbolic decline rate, he (Arps) found out that the decrease in production per unit time as a fraction of the production rate is proportional to a fractional power. The coefficient of his fraction decline when $0 < n < 1.0$ was given as:

$$\boldsymbol{q = \frac{q_i}{(1+nat)^{\frac{1}{n}}}} \tag{5}$$

$$\boldsymbol{N_p = \frac{q_i}{a(1+n)}\left(q_i^{1-n} - q^{1-n}\right)} \tag{6}$$

The coefficient of the decline rate for harmonic decline is unity (n = 1), so the equations become

$$\boldsymbol{q = \frac{q_i}{(1+nat)}} \tag{7}$$

$$\boldsymbol{N_p = \frac{q_i}{a} ln \frac{q_i}{q}} \tag{8}$$

Spivey, et al (1992) Hyperbolic & Harmonic Decline Rates

$$\boldsymbol{Q_{Dd} = \frac{1}{1-b}\left[(1 - 1 + bt_{Dd})^{1-\frac{1}{b}}\right] = \frac{1 - q_{Dd}^{1-b}}{1-b}} \tag{9}$$

$$\boldsymbol{Q_{Dd} = ln(1 + t_{Dd}) = ln\left(\frac{1}{q_{Dd}}\right)} \tag{10}$$

Slider, (1968) presented a simplified type of hyperbolic decline curves analysis. In his analysis he used rate time data. The actual decline curves data were plotted on a transparent paper and compared to a series of semi-log plots of different oilfields cumulative production decline curves with known values of a & n. Slider, (1983), equally produced a tabulated values needed in plotting of hyperbolic type curves using the values of $0.1 < n < 0.9$ with n + 0.1 incremental. He used these in the analysis of production decline cures in order to develop the proper models.

$$\boldsymbol{q = \frac{q_i}{(1+nat)^{\frac{1}{n}}}} \tag{11}$$

$$\boldsymbol{a = \frac{\left({q_i}/{q}\right)^{\frac{1}{n}}}{nt}} \tag{12}$$

Gentry, (1982), prepared a series of plots and different values of the rate exponent (n) ranging from 0 to 1.0 with an incremental value of 0.1. He used the rate with the cumulative oil production and the intervening time to obtain the values of ‘‘n’’ for a production in hyperbolic decline curves.

Gentry, (1986), recommended that conventional decline curves analysis should only be used when the mechanical conditions and the reservoir drainage remain fairly unchanged and the oil-well is produced at steady capacity.

The disadvantages of all these endeavours include: Semi-log type curves miss matched results in wrong modelling and the semi-log plot and/or cross-match is that an exact fit of the data is not easily possible, but the techniques are relatively rapid in use.

Fetkovitch, (1980), designed an advanced decline curves analysis approach, which has been applicable for changes in pressure or drainage. His approach was similar to pressure testing in log-log plots. $\boldsymbol{\frac{q}{q_i}\ \ VS\ \ a\,t}$ and $\boldsymbol{q_{Dd}\ \ VS\ \ t_{Dd}}$. Fetkovitch used different values of ‘‘n’’, in Arps equations and plotted out curves where he concluded that Arps’ equations are only suitable for rate-time depletion data, but in transient time data will result in incorrect forecasts. In the full size type curves, field data were plotted on a tracer paper, which are the same as log-log paper scale as the full-size types curves. The best fit in bbl/unit time would be chosen. A match can be used to obtain values of q_i & q for actual data. These data are then used for appropriate equations to be used in the analysis of the rate-time as well as cumulative hydrocarbons production ($N_p\ or\ G_p$).

Hudson and Nurse, (1985), recommended that the most effective method for reserves estimation is the depletion stage.

C. The Power Law Decline Rate Constant Method

Ilk, et al (2008) presented the ‘‘Power - Law’’ decline method which uses a different functional form of D-Parameter given by:

$$\boldsymbol{D = D_\infty + D_1 t^{-(1-n)}} \tag{13}$$

D is approximated by a decaying power-law function from transient and through transition flow and exhibits a near constant behavior ($\boldsymbol{ie\ D_\infty}$) at very large time. This is contrast to hyperbolic rate decline that leads to a constant behavior at early time and becomes a unit slope power law decaying function at larger times. The advantage of their mathematical equation is that it is flexible enough to cover the transient, transition and boundary dominated flow and to large time reduces an exponential decline ($\boldsymbol{D} = \boldsymbol{D}_\infty$). They then combined their equation with Arps' equation as:

$$\frac{1}{D} = \frac{q}{dq/dt} + D = D_\infty + D_1 t^{-(1-n)} \tag{14}$$

Solving eq. (14) gives:

$$q = q_i\, e^{\left[-D_\infty t - \frac{D_1}{n} t^n\right]} \tag{15}$$

where: $D_1 = Decline\ constant\ when\ t \to \infty$

$n = Time\ exponent$

$q_i = Rate\ intercept\ at\ t = 0$

The difference between their q_i *and* q_i in Arps decline models is because it refers to rate at the onset of stabilized flow, while q_i in Arps decline models refers to flow rate at early stage of a well.

Edwardson, et al (1962) provided the mathematical equation for cumulative hydrocarbons values using dimensionless terms:

When $\mathrm{t_D} < 200$

$$Q_D = \frac{1.12838 t_D^{0.5} + 1.19328 t_D^{1} + 0.27 t_D^{1.5} + 0.086 t_D^{2}}{1 + 0.62 t_D^{0.5} + 0.041301 t_D^{1}} \tag{16}$$

When $\mathrm{t_D} > 200$

$$Q_D = \frac{-4.23\, t_D^{0.5} + 2.026\, t_D}{ln\ t_D} \tag{17}$$

Bruns, (1986) tried, using fractions as $\frac{1}{2}$, $\frac{5}{8}$ *and* $\frac{3}{4}$ in his dimensionless time-function and found out that using $\frac{1}{2}$ reduces the discontinuity between the transient streams and hyperbolic streams.

Spivey, et al (1992) provided detailed equations for generating the transient and boundary dominated streams of the cumulative hydrocarbons production type curves. They found out that the transition from transient equation to boundary dominated flow equation for cumulative hydrocarbons occurs at $t_{Dd} = 0.6$, compared to Fetkovitch's dimensionless flow rate type curves, where the transition occurs at $t_{Dd} = 0.1$. His work showed also that the type curves of cumulative hydrocarbons production can be obtained with their derivative using semi-log to give a set of type curves that uses only cumulative hydrocarbons production data and net rate. His plots tend to have less scatter points than the traditional Fetkovitch's type curves. Their derivative $\frac{\mathrm{d(Q)}}{\mathrm{d(ln\, t)}}$ is equivalent to the traditional derivative qt and in dimensionless form as: $\mathbf{q_{Dd}} \cdot \mathbf{t_{Dd}}$. They plotted $q_{Dd} t_{Dd}\ vs\ Q_{Dd}$.

Johnson and Bollens, (1945) defined the loss-ratio and the derivative of loss-ratio function as:

$$q = q_i\, e^{-\left[D_\infty t + \frac{D_1}{n} t^n\right]} = q_i\, e^{-[D_\infty t + D_i t^n]} \tag{18}$$

where

q_i = Rate at $t = 0$ (called rate intercept)

D_1 = Decline rate constant intercept at $t = 1\ day$

D_∞ = Decline rate constant at t = infinity (∞)

n = Time exponent and t = Time & $D_i = \frac{D_1}{n}$

D. Concept of Integral Type Curves

Blasingame, et al (1989) introduced the concept of integral type curves in the well testing fields. Spivey et al (1992) extended Blasingame and his students' work concept to decline curves analysis. In their work they stated that the hydrocarbons production data are usually very noisy (disarranged). Plotting a rate–integral or cumulative hydrocarbons production should reduce the noise and would make the data much more analyzable. The rate-integral is related to the cumulative hydrocarbons production as defined in the following equations.

$$q_i = \frac{Q}{t} \tag{19}$$

$$q_{Ddi} = \frac{Q_{Dd}}{t_{Dd}} \quad dimensionless \tag{20}$$

Where:

$\mathrm{q_i} = \mathrm{q_{Ddi}}$ = Initial production rate

$\mathrm{Q} = \mathrm{Q_{Dd}}$ = cumulative rate & $\mathrm{t} = \mathrm{t_{Dd}}$ = time

The dimensionless term is obtained by dividing the cumulative hydrocarbons production by the time of flow. The rate-integral has a direct physical interpretation, as average hydrocarbons production rate from the beginning of production to the current time (actual stage).

E. Advantages in Their Work

Decline techniques are not limited to constant bottomhole flowing pressure like those in Arps and Fetkovitch. Decline techniques account for variations in bottomhole flowing pressure in the transient regime. In addition their analysis can work fine in the changing values of reservoir PVT properties with the changing reservoir pressure for both oil and gas. The method uses superposition time function that only requires one depletion stem for type curves matching. When the type curves are plotted using Blasingame's superposition time function the analytical exponential stem of Fetkovitch's type curves becomes harmonic. The significance of this is that if the inverse of this flowing pressure is plotted against time pseudo steady state depletion at constant flow rate follows a harmonic decline. In effect it allows depletion at a constant pressure to appear as pseudo steady state depletion at constant rate, provided that the rate and pressure decline monotonically. Blasingame improved Fetkovitch's decline curves analysis by the introduction of two additional type curves, which are plotted concurrently with the normalized rate type curves. The rate integral and rate-integral derivative type curves aid in obtaining a more unique match. The derivation of the data obtained when both the rate and the flowing pressure are varying can now be analyzed if the material balance time is used instead of actual production time. This is possible, because an exponential decline would be the harmonic decline stem (q_{Dd} vs t_{Dd}) is exponential and ($\frac{q}{\Delta p}$ vs $\frac{Q(t)}{q(t)}$) is harmonic. They developed type curves which showed the analysis of transient stems alongside with the analytical harmonic decline, but with the rest of the empirical hyperbolic stems absent.

Johnson and Bollens, (1928), used power law and analyzed the loss-ratio and loss-ratio derivatives function. They showed that the derivative equation is flexible enough to cover transient, transition and boundary dominated flow with large time reduces to an exponential decline.

F. *Fractional Hydrocarbons Decline Rate*

Arps, (1945) explained an exponential oil or gas decline rate using a straight line graph that could be extrapolated to initial state of a reservoir conditions. He stated that the data suitable for used in the prediction must satisfy a constant fractional drop in the reserves production. In His work the value of decline exponent used was zero (n = 0). His mathematical model equations are:

$$\frac{q_i}{qdt} = -aq = constant \quad or \quad a = \frac{dq_i/q}{dt} \tag{21}$$

$$N_p = \frac{q_i - q}{a} \quad and \quad a = \frac{q_i - q}{N_p} \tag{22}$$

In the hyperbolic decline rate analysis the decrease in oil or gas production per unit time as a fraction of the production rate is proportional to a fraction power called ''n''. The fractional power range was given as: $0.1 < n < 0.9$. Arps, stated that the most efficient data for this type of hydrocarbons production decline curves are the oilfield depletion data. His mathematical definitions were:

$$q = \frac{q_i}{(1 + na_1 t)^{\frac{1}{n}}} \tag{23}$$

$$N_p = \frac{q_i}{a_1(1-n)}\left[q_i^{1-n} - q^{1-n}\right] \tag{24}$$

In the harmonic fractional hydrocarbons decline rate, the type curves for oil or gas production decline rate are similar to hyperbolic decline rate determination methods, in that the slope on the semi-log plot decreases with time, but for a harmonic oil or gas production decline rate the decrease in production per unit time is a fraction of the production rate which is directly proportional to the rate. This is observed in reservoir flow dominated by gravity drainage. The fraction power, n = 1 and the mathematical equations he used were:

$$q = \frac{q_i}{1 + na_1 t} \tag{25}$$

$$N_p = \frac{q_i}{a_1} ln\left(\frac{q_i}{q}\right) \tag{26}$$

where

a = Actual decline fraction of production rate

t = Time unit (s, hr or yr)

a_1 = Initial oil or gas production decline

G_p = Cumulative gas production in a time, t (Mscf)

N_p = Cumulative oil production in a time, t (stb)

q_i = Initial production rate, scf/ stb/unit time

q = actual rate per unit time, scf/stb/time

n = Rate exponent or

$\frac{1}{n} = h$ = hyperbolic decline constant

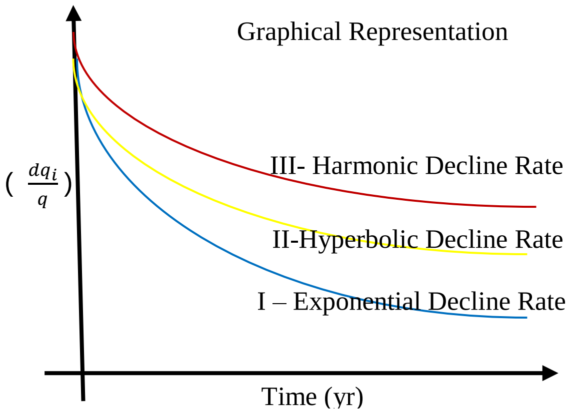

Figure 1. Arps' Oil Production Decline Rate Curves

Slider, (1968), recommended that if the data are doubted a mathematical model of an actual cumulative oilfield production data be plotted to determine the percentage fitness with a range $90 < R^2 < 100\%$. The deviation of the model used in generating the graph must generate a line of best fit for the oilfield reserves estimation history. Slider added that the most effective reserves estimation using conventional methods must be determined at the best fitted in the semi-log hyperbolic plots or log-log hyperbolic plots. He stated that the best fitted curves must be suitable for extrapolating to the oilfield reserves initially in place conditions at the reserves point.

G. *Fractional Decline Exponent Conventionally*

Field experience showed that fractional decline exponent would be zero (n = 0) in the case of:

Single phase liquid, high pressure gas, Tubing and choke restricted gas and Poor water flooding performances in the production system.[Spivey, et al, 1992]

Higher fractional decline exponent value of ($\mathbf{0} < n < 1$) in the case of: Production under solution gas drives, the lower the relative permeability the smaller is the gravity of gas produced, hence the decline rate of the reservoir is slower and accordingly the production decline is lower with high value of decline exponent ($\boldsymbol{n} > 1$). [Ramsay and Guenero, 1969]

- Simulation studies for the range of oil and gas relative permeability (K_{ro} and K_{rg}) have shown that the decline exponent (n) ranges from 0.1 to 0.4 ($\mathbf{0.1} < n < \mathbf{0.4}$), giving an average value of 0.3, but the production data above bubble point pressure are not analyzed with the data below the bubble point, because decline analysis valid when the recovery mechanism and the operating conditions do not change with time. Above the bubble point pressure, n = 0 and the decline rate is constant. Below the bubble point pressure the decline const (n) increases as in the solution gas drive condition. For gas wells $\mathbf{0.4} < n < \mathbf{0.5}$ or average of n = 0.45 and conventionally light oil reserves under edge water drive (effective water drive), n = 0.5. [Gentry and McCray, 1978]

- In pressure maintenance system such as gas & water injections, active-water drive, and gas-cap expansion drive, where the hydrocarbons are saturated the production rate would remain fairly constant. And the decline tends to zero small reservoir pressure decline leads to high production driving force with a corresponding small production decline rate. In this effect the decline rate constant is theoretically greater than unity ($\boldsymbol{n} > 1$). Much later when the oil column thins, the production rate would decline exponentially with $\boldsymbol{n} = \mathbf{0}$ and the hydrocarbons production is replaced by water. [Blasingame, et al, 1989]
- In commingled layered reservoirs n lies between 0.5 and 1.0. Decline analysis is best initialized from the start of the decline rate. He added that it is possible under certain production and scenarios that initially the rate does not decline. [Fetkovitch, 1984]

H. Natural Reservoirs Oil/Gas Production Decline

This section discusses the theory of natural oilfield production decline types and modelling. There are 3 basic types, theoretical, semi-theoretical and empirical models that can be used to explain the phenomenon of oilfield hydrocarbons depletion and their models development. An oilfield is one of the natural resources or commodity that is finite and not renewable. [Lantz, 1971].

I. Decline Rate correlation as function of Time

Poston, (1998), worked on oil and gas production rate decline as a function of time. He found out that the loss of a reservoir pressure or the changing relative volumes of the produced fluids are usually the cause of the rate decline with time. He concluded that a production history may vary from a straight line to a concave up-ward curve, but in any case the objective of decline curve analysis is to model the production history with the equation of a line. Table 1 summarizes Poston's model equation using a line to forecast future hydrocarbons production. He expressed the exponential decline rate in two basic forms:

TABLE I. POSTON PRODUCTION FORECAST MODEL

Log-Rate-Time shape Name	Model	Decline Trend
Straight line - Exponent	-	Stepwise
Straight line - ,,	Arps	Continues straight
Converging - Hyperbolic	,,	Continues curves
Limited curves - Harmonic	,,	Un-converge curves
Un-converging - Amended	-	Dual-infinity action to limited curves

- Effective or Constant Percentage Decline

This decline showed the incremental rate loss concept in mathematical terms as a stepwise function. Table 2 shows the effective and continues (normal) exponential equations.

- Normal or Continues Rate Decline

This showed the negative shape of curves representing hydrocarbon production rate versus time for oil and gas reservoirs. His equations showed the relationship between normal and effective decline rates. $D = -ln(1-d)$ and conventionally assumes the decline in percentage of a year (% yr). Table 2 this shows details. $d = 0$, For exponential case, $0 < d < 1$, Hyperbolic case and $d = 1$, Harmonic case

TABLE II. EFFECTIVE AND CONTINUES EXPONENTIAL

Action	Constant Rate	Continues Rate
Decline Rate	$\boldsymbol{d} = \frac{\boldsymbol{q_1} - \boldsymbol{q_2}}{\boldsymbol{q_1}}$	$\boldsymbol{D} = \frac{\boldsymbol{ln}(q_1/q_2)}{\boldsymbol{t}}$
Production Rate	$\boldsymbol{q_2} = \boldsymbol{q_1}(\mathbf{1}-\boldsymbol{d})^t$	$\boldsymbol{q_2} = \boldsymbol{q_1}\boldsymbol{e}^{-(\boldsymbol{Dt})}$
Time elapsed	$\boldsymbol{t} = \frac{\boldsymbol{ln}(q_2/q_1)}{-\boldsymbol{ln}(\mathbf{1}-\boldsymbol{d})}$	$\boldsymbol{t} = \frac{\boldsymbol{ln}(q_1/q_2)}{\boldsymbol{D}}$
Cumulative value	$Q_p = \frac{q_1 - q_2}{-ln(1-d)}$	$\boldsymbol{Q_p} = \frac{\boldsymbol{q_1} - \boldsymbol{q_2}}{\boldsymbol{D}}$

J. Poston's Curves Characteristics Conclusion

- Rate-time curves tend to a downward manner
- The semi.log rate-time curve is a straight line in exponential decline equation while hyperbolic and harmonic decline are curved lines
- The Cartesian rate-cumulative recovery plots are straight lines for exponential, hyperbolic and harmonic or curved
- A semi-log rate-cumulative production plots are straight line for harmonic while exponential and hyperbolic decline rates are curved. Fig 2 below shows the types of decline rates.
- Harmonic tends to flatten out with time.

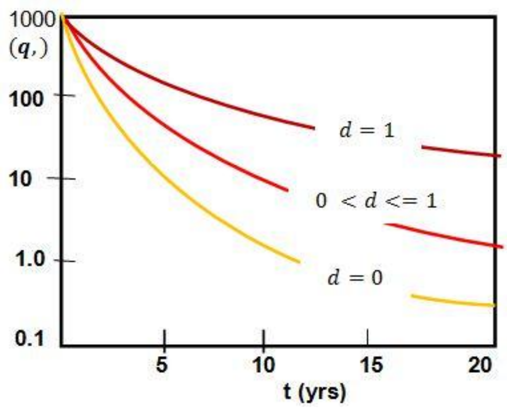

Figure 2. Poston's Production Decline Trend

Theoretically exponential constant (d) varies in the positive (+ve) or negative (-ve) manner. The –ve values indicate an increasing production rate while the +ve value implies infinite, hence cumulative production must be infinite for $d \geq 1$. This statement shows why exponential term cannot be greater than unity. His study indicated that exponential decline must vary over a large decline constant ($0 < d < 1$)

K. Well Production Performance

Golan and Whitson, (1986), defined production decline analysis as a traditional means of identifying well production problems and predicting a well performance with respect to its life based on real production data. They used empirical decline models that have little fundamental justification, similar to those of Arps'. The general model is the hyperbolic decline while others are degeneration of hyperbolic decline model. These models are related through relative decline rate with mathematical equation as:

$$\frac{1}{q}\frac{dp}{dt} = -bq^{d} \quad (27)$$

where

b and d = Empirical constant decline based on production data. When d = 0, the equation degenerates to an exponential decline model. When d = 1, it yields a harmonic decline model and when $0 < d < 1$, the equation yields a hyperbolic decline model. They recommended their model for both used in oil and gas wells.

L. Relative Decline Rate

Economides, et al (1994), considered an oil well drilled in a volumetric oil reservoir where they assumed that the wells production rate starts to decline when a critical (lowest permissible) bottom hole pressure (BHP) is reduced. Under the pseudo-steady-state flow condition the production rate at a given decline time (t) can be expressed mathematically as:

$$q = \frac{k\,h\,(P_t - P_{wf})}{141.2\,B_o\,\mu\,ln\left(\frac{0.472\,r_e}{r_w}\right) + S} \quad (28)$$

$$N_p = \int_0^t \frac{k\,h\,(P_t - P_{wf})\,dt}{141.2\,B_o\,\mu\,ln\left(\frac{0.472\,r_e}{r_w}\right) + S} \quad (29)$$

$$N_p = \frac{C_t N_i}{B_o}(P_0 - P_t) \quad (30)$$

where

P_t = Average pressure at decline time, t

P_{wf} = Critical BHP at production decline

C_t = Total reservoir compressibility

N_i = Initial oil in place in the well drainage area

P_0 = Average pressure at decline time zero

Edwardson, (1962), provided detailed equations for generating the transient and the boundary dominated streams of the cumulative production type curves. He stated that the transient flow rate and cumulative productions are reported in dimension form q_D and Q_D respectively as function of dimension time, t_D.

Mathematically as:

$$Q_D = \frac{-4.29881 + 2.02566\,t_D}{ln\,t_D} \quad (31)$$

The well test based on Q_D and t_D are converted to decline based on Q_{Dd} and t_{Dt}

$$Q_{Dd} = \int_o^{t_{Dd}} q_{Dd} t_{Dd} = \frac{Q_D}{\frac{1}{2}\left[\left(\frac{r_e}{r_w}\right)^2 - 1\right]} \quad (32)$$

$$t_{Dd} = \frac{t_D}{\frac{1}{2}\left[\left(\frac{r_e}{r_w}\right)^2 - 1\right]\left[ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4}\right]} \quad (33)$$

$$Q_{Dd} = 1 - e^{-t_{Dd}} = 1 - q_{Dd} \text{ Exponential} \quad (34)$$

$$Q_{Dd} = \frac{1 - q_{Dd}^{1-b}}{1-b} \quad \text{(Hyperbolic)} \quad (35)$$

$$Q_{Dd}\,ln(1 + t_{Dd}) = ln\left(\frac{1}{q_{Dd}}\right) \text{ Harmonic} \quad (36)$$

Amini, et al, (2007), used elliptical flow to govern flow regime in a low permeability gas reservoir with elliptical outer binding. He described these cases as one production from an elliptical wellbore, elliptical fracture or a circular wellbore in an anisotropic reservoir system, which can be considered to be an elliptical inner boundary. They stated that an elliptical reservoir surrounded by an elliptic aquifer is an elliptical outer boundary. They also stated that the reservoir is assumed to be a single-layer system that is isotropic, horizontal and uniform thickness and constant flow rate. Mathematically:

$$q_D = \frac{141.2\,B\mu q}{K\,h\,\Delta P} \quad (37)$$

$$K = \frac{141.2\,B\,\mu}{h}\left[\frac{q/_{\Delta P}}{q_D}\right] \quad (38)$$

Agarwal and Gardner, (2008), presented new decline type curves for analyzing production data. Their method builds on Fetkovitch's and Palacio-Blasigame's ideas. They utilized the concept of the equivalence between constant rate and constant pressure solution. They also presented new type curves with dimensionless variables based on the conventional well-test definition as in Fetkovitch and Blasigame. They equally included primary and semi-log pressure derivatives plots (decline analysis inverse formant). They as well presented rate versus cumulative and cumulative versus time plots. Rate – cumulative Production analysis mathematically:

$$Q_{DA} = \frac{t_{DA}}{P_D} = q_D t_{DA} \quad (39)$$

$$q_D = \frac{141.2\,qB\mu}{K\,h\,(P_t - P_{wf})} \quad (40)$$

Wattenbarger, (1998), observed long linear flow in many gas wells. These were very tight reservoir with hydraulic fractured boundary of the well. Wattenbarger presented new types curves to analyze the production data of gas wells. He assumed a hydraulically fractured well in the centre of a rectangular reservoir. The fracture was assumed to be extended to the boundaries of the reservoir. Figure 3 shows his sketch.

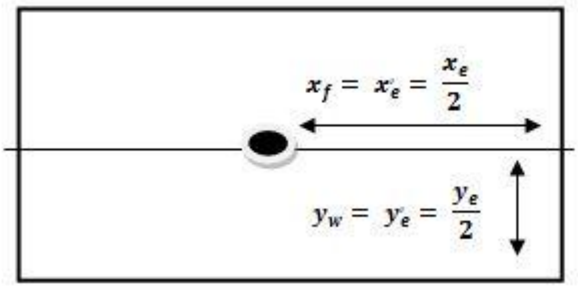

Figure 3. Constant rate in closed reservoir solution

Mathematically:

$$P_D = \frac{\pi}{2}\left(\frac{y_e'}{x_f}\right)\left[\frac{1}{3}+\left(\frac{x_f}{y_e'}\right)^2 t_{Dd}\right] - \frac{2}{\pi^2}\left(\frac{y_e'}{x_f}\right)\sum_{x=1}^{\infty}\frac{1}{n^2}e^{\left[n^2\pi^2\left(\frac{x_f}{y_e'}\right)^2 t_{Dd}\right]} \quad (41)$$

Agarwal, et al, (2007), explained the importance of water influx in gas reservoir. They observed that an appreciably water influx in a gas reservoir acts as pressure maintenance naturally delaying the decline initiation. The benefit is that much of the hydrocarbons are produced. The disadvantage is that such a reservoir is difficult to model, due to less knowledge of the aquifer behavior and life span.

King-Hubbert and Robertson, (2004), suggested in their work ‘‘Modified Hyperbolic Decline’’ that at some point in time the hyperbolic decline is converted into an exponential decline. They extrapolated hyperbolic decline over long periods of time and found out that it frequently results in unrealistically high pressure. To avoid this problem, they made their suggestion. They assumed that for a particular example, the decline rate (D) starts at 30% of flow and decline through time in a hyperbolic manner. When it reaches a specified value say 10% of the hyperbolic decline can be converted to an exponential decline and the forecast continued using the exponential decline rate of 10%. Fig 4 shows the graphical representation of their work:

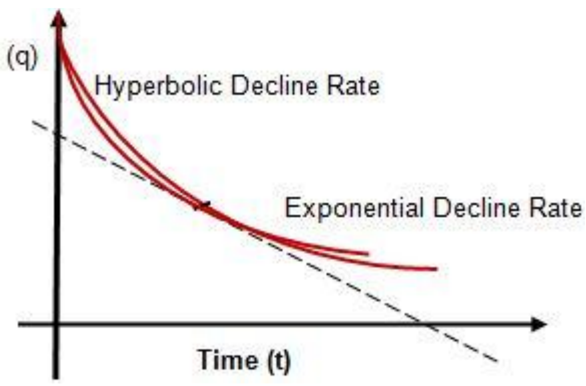

Figure 4. Hyperbolic to Exponential Decline Trend

Mathematically:

$$q = q_i \frac{[(1-B)^b e^{-Dt}]}{1-B\, e^{(-Dt)^b}} \quad (42)$$

$$Q = q_i \frac{1-B}{BD}\left[1-\frac{1}{1-B\, e^{(-Dt)^{b}-1}}\right] \quad (43)$$

When b = 1

$$Q = q_i \frac{1-B}{BD}\, ln\left[1-\frac{1-Be^{-Dt}}{1-B}\right] \quad (44)$$

Or

$$D = \frac{D_i}{q_i^{b-1}(1+bD_it)^{\frac{1}{b}}} \quad (45)$$

$$q = q_i(1+bD_it)^{-\frac{1}{b}} \quad (46)$$

Ramsay and Guerrero, (2002), Study also included relative decline rate and they indicated in their work that about 40% of leases have $b > 0.5$ and commingled layered reservoirs fall between $0.5 < b < 1.0$.

II. Research Methodology

A. *Models Development Procedure*

Two principal methods of postulating the evaluation models were used, the projectile and parabolic dominated hydrocarbons flow regimes.

B. *Evaluation Model-I: The Projectile Fluid Flow*

The projectile flow is common in natural depletion of hydrocarbons deposits from the initial stage to an abandonment stage. In this case a plot was used to study the complete depletion from the initial state to the transient state, steady state and decline state. Energy building up starts from time, t_o to time, t_1 in fig 2.1 and Fig 2.2. The steady state flow (called the plateau) starts from time, t_1 to time, t_2, after this the rate decline state sets in with or without transition state, from time, t_2 to time, t_3 covering the total or cumulative gas or oil recovery value (in scf or stb). Any recovery from time, t_3 to time, t_f covers the hydrocarbons supposed be the residual oil or gas of that reservoir. The complete depletion of the hydrocarbons in that reservoir (called hydrocarbons initially in place) is from time, t_o to time, t_f. The equation of the area of that shape (trapezium) is the value of the hydrocarbons initially in place. Fig 5 and Fig 6 show more of this. This is only obtainable in theory for reserves estimation, so it is an extrapolated value.

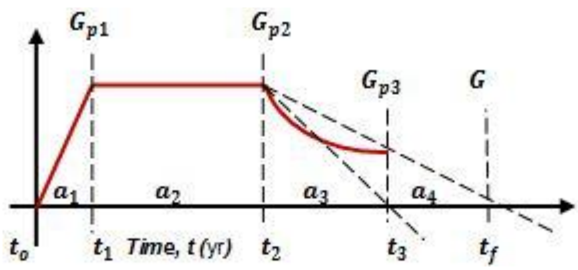

Figure 5. Schematic of Gas Flow during Production

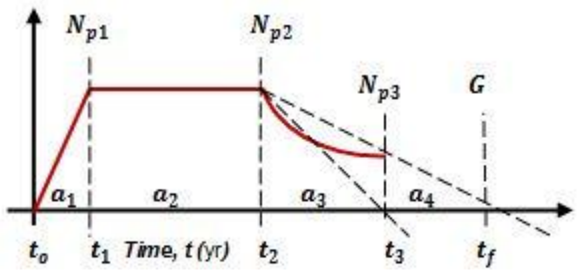

Figure 6. Schematic of Oil Flow during Production

1) Assumptions:

- An oilfield must contain a reserve initially in place (N), which reduces per unit time, due to hydrocarbons production operations.
- The flow rate (q) of oil stream production continues to change from time, t_o to time, t_1 and from time, t_1 to time, t_2 and from time, t_2 to time, t_3, (Fig 5 or Fig 6), so that time, t_f could be extrapolated.
- The hydrocarbons production (N_p) per unit time declined from the initial value to minimum, dN/dt.
- The quantity of the reserves remaining in the reservoir is N_f. The general equation for natural production of an oilfield reserves is given as eqn. 47 and eqn. 48:

$$\begin{bmatrix} Actual \\ Reserves \\ Produced \end{bmatrix} = \begin{bmatrix} Reserves \\ Initially \\ in\ Place \end{bmatrix} - \begin{bmatrix} Actual\ Reserves \\ remaining\ in\ Place \end{bmatrix}$$

$$\boldsymbol{G_p} = \boldsymbol{G} - \boldsymbol{G_f} \quad (47)$$

$$\boldsymbol{N_p} = \boldsymbol{N} - \boldsymbol{N_f} \quad (48)$$

Using Fig. 5, the actual gas reserves produced in a given time and gas initially in place are expanded as:

$$[Gas\ Produced] = [Area\ of\ the\ Trapezium]$$

$$= \frac{1}{2}[Sum\ of\ the\ Parallel\ Sides] * [Height]$$

$$G_p = \frac{q_i}{2}[(t_2 - t_1) + (t_3 - t_0)] \quad (49)$$

Or

$$[Gas\ Produced] = [Area\ of\ the\ Trapezium]$$

$$[Gas\ Produced] = [a_1 + a_2 + a_3]$$

$$a_1 = Area\ of\ \Delta AHO$$

$$a_1 = \frac{q_i}{2}[t_1 - t_0] \quad (50)$$

$$a_2 = Area\ of\ the\ rectangle\ ABFH$$

$$A_2 = q_i[t_2 - t_1] \quad (51)$$

$$a_3 = Area\ of\ \Delta BEF$$

$$a_3 = \frac{q_i}{2}[t_3 - t_2] \quad (52)$$

Adding up eqn. 50, 51 and 52 gives eqn. 53

$$G_p = \frac{q_i}{2}[(t_2 - t_1) + (t_3 - t_o)] \quad (53)$$

or

Using the equation of the curve part of Fig. 5/6

$$a_3 = [Gas\ recovered\ in\ Decline\ rate\ Stage]$$

2) Projected Hydrocarbons Production

$$G_p = \frac{[q_i - q]}{b} \quad or \quad qt \quad (54)$$

$$N_p = \frac{[q_i - q]}{b} \quad or \quad qt \quad (55)$$

3) Hydrocarbons Production Models

The general equation for natural production of an oilfield reserves is the product of the rate-constant and the actual rate raised to power-n. This is given by parabolic flow regime (eqn. 56):

$$\begin{bmatrix} Production \\ Rate\ Chane \\ with\ Time \end{bmatrix} = \begin{bmatrix} Decline\ Rate \\ Constant \end{bmatrix} \begin{bmatrix} Actual\ Rate\ in \\ n - order \end{bmatrix}$$

$$\frac{dq}{dt} = -bq^n \quad (56)$$

Using the curve in Fig 5/Fig 6 and eqn. 56, the actual oil or gas production rate in a given time is postulated as follows: When n = 1 is a $\mathbf{1^{st}}$ order decline rate parabolic flow:

$$\int_{q_i}^{q} \frac{dq}{q} = -b \int_{o}^{t} dt \quad (57)$$

Solving eqn. 57 gives, the governing equation, eqn. 58

$$lnq - lnq_i = -bt \quad (58)$$

The governing equation, eqn. 58 is used to obtain hydrocarbons production rate (q) by removing the log in eqn. 58 and rearranging gives eqn. 59. To estimate the rate-constant (b), eqn. 58 is rearranged to obtain eqn. 60.

$$\boldsymbol{q} = \boldsymbol{q_i e^{-bt}} \quad (59)$$

$$\boldsymbol{b} = \boldsymbol{\frac{ln(q_i/q)}{t - t_i}} \quad (60)$$

4) Cumulative Hydrocarbons Production Model

The general equation for natural production of an oilfield reserves is the product of the hydrocarbons flow rate and the actual time elapsed. This is given by parabolic flow regime (eqn. 61 and eqn. 62):

$$\begin{bmatrix} Cumulative \\ Oil\ or\ Gas \\ Produced \end{bmatrix} = \begin{bmatrix} Production \\ Rate\ Per \\ Unit\ Time \end{bmatrix} \begin{bmatrix} Actual \\ Time \\ Elapsed \end{bmatrix}$$

$$\boldsymbol{G_p} = \boldsymbol{qdt} \quad (61)$$

$$\boldsymbol{N_p} = \boldsymbol{qdt} \quad (62)$$

Using Fig 5 or Fig 6, eqn. 61 or eqn. 62, the actual gas or oil cumulative production at a given time is postulated as follows:

$$\boldsymbol{G_p} = \boldsymbol{\int_o^t qdt} \quad (63)$$

$$N_p \quad = \quad \int_o^t q dt \tag{64}$$

But $q = q_i e^{-bt}$ in eqn. 59 substituting this in eqn. 63 gives eqn. 65, the cumulative gas production and in eqn. 64 gives eqn. 66, the cumulative oil production.

$$G_p = \int_o^t q_i e^{-bt} dt \tag{65}$$

$$N_p = \int_o^t q_i e^{-bt} dt \tag{66}$$

Solving eqn. 65 gives eqn. 67, the governing equation for gas cumulative production and solving eqn. 66 gives eqn. 68, the governing equation for actual oil cumulative production.

$$G_p = \frac{q_i}{b}\left[1 - e^{-bt}\right] \text{ For gas systems} \tag{67}$$

$$N_p = \frac{q_i}{b}\left[1 - e^{-bt}\right] \text{ For oil systems} \tag{68}$$

This implies that the projected production is:

$$a_3 = G_{p3} = \frac{q_i}{b}\left(1 - e^{-bt}\right) \tag{69}$$

Similarly:

$$N_{p3} = \frac{q_i}{b}\left(1 - e^{-bt}\right) \tag{70}$$

Summing up eqns. 50, 51 and 69 gives eqn. 71:

$$G_p = q_i\left[t_2 - 0.5(t_1 + t_o) + \frac{(1 - e^{-bt_3})}{b}\right] \tag{71}$$

Equations 49, 53 and 71 are the gas production decline analysis models evaluation equations postulated.

5) Hydrocarbons Initially in Place (G or N) Postulation

$$\begin{bmatrix} Actual\ Gas\ Reserves \\ Initially\ in\ Place \end{bmatrix} = \begin{bmatrix} Area\ of\ the \\ Trapezium\ OABDO \end{bmatrix}$$

$$[Gas\ in\ Place, G] \quad = \quad [Area\ \ OABDO]$$

$$[Area\ \ OABDO] = \frac{q_i}{2}\left[(t_2 - t_1) + (t_f - t_o)\right]$$

$$G = \frac{q_i}{2}\left[(t_2 - t_1) + (t_f - t_o)\right] \tag{72}$$

Equation 72 is the actual gas initially in place (GIIP). This is very possible since gas production is the product of the flow rate, q and time, t ($G = q * t$). Similarly Using Fig 6, the actual oil reserves produced in a given time and the actual oil initially in place were postulated in the same procedure:

$$N_p = \frac{q_i}{2}\left[(t_2 - t_1) + (t_3 - t_0)\right] \tag{73}$$

$$N_p = q_i\left[t_2 - 0.5(t_1 + t_o) + \frac{1}{b}\left(1 - e^{-bt_3}\right)\right] \tag{74}$$

Equations 49, 73 and 74 are the gas production decline analysis models evaluation equations postulated.

$$N = \frac{q_i}{2}\left[(t_2 - t_1) + (t_f - t_o)\right] \tag{75}$$

Equation 75 is the actual oil initially in place (OIIP). This is very possible since oil production is the product of the flow rate, q and time, t (yr).

C. Evaluation Model – II Parabolic Fluid Flow

The dome shape of Fig 2.3 indicates a parabolic flow rate from lowest at point-P to a maximum point – Q and declines to abandonment at point – R. The curve can be extrapolated from point - R to point – T, for estimation of oil or gas initially in place. In the case of Fig 2.4 the reservoir pressure is just slightly above the bubble point or at bubble point pressure. The implication of this case is that decline starts right from the early stage of production at point –Y to point - Z. The curve can be extrapolated from point - Z to point – X, for estimation of oil or gas initially in place.

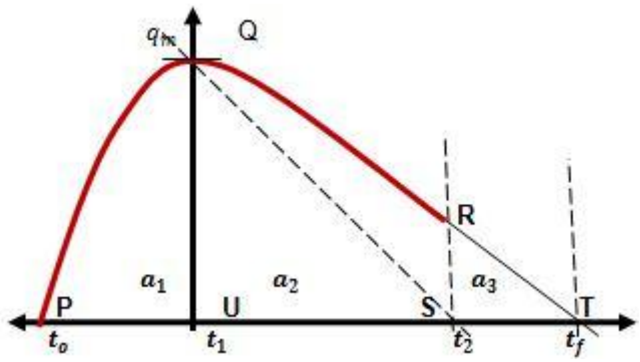

Figure 7. Schematic of Oil/Gas in Parabolic Flow Regime

In this case the reservoir started by building up the internal energy for some time from time, t_o to time, t_1 in fig 7, because the reservoir was fairly saturated, so failed to attain boundary dominated flow at initial state. Instead it built-up from the initial stage to the transient and transition stage at point – Q, but the flow period was too short. To this effects steady state flow (called the plateau) was not observed in the curve at time, t_1 instead rate decline state sets in from time, t_1 to time, t_2. After this the rate decline state sets in with or without transition state, from time, t_2 to time, t_f covering the total or cumulative gas or oil recovery value (in scf or stb). Any recovery from time, t_2 to time, t_f covers the hydrocarbons supposed to be the residual oil or gas of that reservoir. The complete depletion of the hydrocarbons in that reservoir (called hydrocarbons initially in place) is from time, t_o to time, t_f. The equation of the area of that shape (trapezium) is the value of the hydrocarbons initially in place (Fig 7). This is only obtainable in theory for reserves estimation, so it is an extrapolated value.

1) Hydrocarbons Production per Unit Time (stb/yr) Model

$$\begin{bmatrix} Total\ Hydrocarbons \\ Production\ per\ Time \end{bmatrix} = \begin{bmatrix} Area\ of \\ Curve, a_1 \end{bmatrix} + \begin{bmatrix} Area\ of \\ Curv, a_2 \end{bmatrix}$$

$$G_p = \frac{q_i}{2}\left[(t_1 - t_0) + (t_2 - t_1)\right] \text{ For Gas} \tag{76}$$

$$N_p = \frac{q_i}{2}\left[(t_1 - t_0) + (t_2 - t_1)\right] \text{ For Oil} \tag{77}$$

2) Hydrocarbons Initially in Place, stb (Fig 7) Models

$$\begin{bmatrix} Initial \\ Hydrocarbons \\ in\ Place \end{bmatrix} = \begin{bmatrix} Area\ of \\ Curve, a_1 \end{bmatrix} + \begin{bmatrix} Area\ of \\ Curv, a_2 + a_3 \end{bmatrix}$$

$$\boldsymbol{G} = \frac{q_i}{2}\left[(\boldsymbol{t_1} - \boldsymbol{t_0}) + (\boldsymbol{t_f} - \boldsymbol{t_1})\right] \text{ For Gas} \quad (78)$$

$$\boldsymbol{N} = \frac{q_i}{2}\left[(\boldsymbol{t_1} - \boldsymbol{t_0}) + (\boldsymbol{t_f} - \boldsymbol{t_1})\right] \text{ For Oil} \quad (79)$$

Parabolic: *no Observable Transient or Transition*

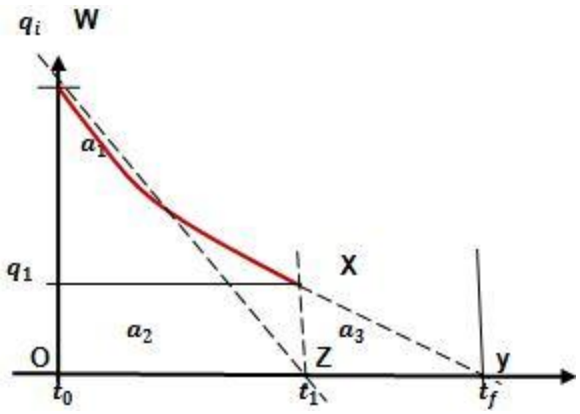

Figure 8. Schematic of Oil/Gas in Parabolic Flow Regime

In this case the oil well flow was on steady state. The boundary conditions were felt right from the start. If the reservoir is not externally supported, it may be difficult to deplete the reservoir completely.

3) Cumulative Production Evaluation Model Using Fig 8

$$\begin{bmatrix} Total\ Hydrocarbons \\ Production\ per\ Time \end{bmatrix} = \begin{bmatrix} Area\ of \\ Curve, a_1 \end{bmatrix} + \begin{bmatrix} Area\ of \\ Curv, a_2 \end{bmatrix}$$

$$\boldsymbol{G_p} = \frac{q_i}{2}[\boldsymbol{t_1} - \boldsymbol{t_0}] \ \boldsymbol{or} \ \boldsymbol{G_p} = \frac{q_i}{b}\left[\boldsymbol{1} - \boldsymbol{e^{-bt}}\right] \quad (80)$$

$$\boldsymbol{G_p} = \frac{q_i - q_1}{2}[(\boldsymbol{t_1} - \boldsymbol{t_0})] + (\boldsymbol{q_1} - \boldsymbol{q_0})(\boldsymbol{t_f} - \boldsymbol{t_1}) \text{ Gas} \quad (81)$$

$$\boldsymbol{N_p} = \frac{q_i - q_1}{2}[(\boldsymbol{t_1} - \boldsymbol{t_0})] + (\boldsymbol{q_1} - \boldsymbol{q_0})(\boldsymbol{t_f} - \boldsymbol{t_1}) \quad (82)$$

4) Oil/Gas Initially in Place Evaluation Model

$$\begin{bmatrix} Total\ Hydrocarbons \\ in\ Place\ initially \end{bmatrix} = \begin{bmatrix} Area\ of \\ Curve, a_1 \end{bmatrix} + \begin{bmatrix} Area\ of \\ Curv, a_2 + a_3 \end{bmatrix}$$

$$\boldsymbol{G} = \frac{q_i}{2}\left[\boldsymbol{t_f} - \boldsymbol{t_0}\right] \quad \text{For Gas} \quad (83)$$

$$\boldsymbol{N} = \frac{q_i}{2}\left[\boldsymbol{t_f} - \boldsymbol{t_0}\right] \quad \text{For Oil} \quad (84)$$

This research work focuses on hydrocarbons production decline rate projection, hydrocarbons cumulative production and hydrocarbons initially in place estimation. The primary data used were the early production rate to project future rates in a given time piece. The values were used to plot curves and the generated curves were empirically used to build the models. In a case where the production data were fairly enough to take care of the build-up flow rate, the steady state (plateau) rate and the decline flow rate, the field data were used directly to generate the curves. The advantage of using projectiles and parabolic methods in model development is that such a model is very flexible. The models could be applied with high accuracy right from the initial reservoir stage, through the transient stage, transition stage to the decline rate stage. Empirically observant and tactfully the models could be used in an induced hydrocarbons production operation. If the user is not empirically observant enough, he has to use fluids displacement methods in the induced recovery operations. The disadvantage is that the models do not take care of pressure drawdown, so the projected rate decline trend or cumulative hydrocarbons production trend depend on pressure sustainability.

D. Evaluated Model Equations Applications

This section presents the application of the models equations using regional and generic data. The models were applied in a gas well which had produced 639,164.26MMM scf in 22½ years, showed a cumulative gas production of 638.274.38 MM scf, with percentage accuracy of 99.86%, comparable to the field production records. The models were equally used to estimate gas Initially in Place (GIIP) value. (698,541.00MMscf).

The models were applied again in the Delta State South Oilfield, which started in March, 1968 to March, 1978 with Cumulative oil production of 16,651.1MStb. The results showed a cumulative oil production value of 16,877M stb and estimated Oil initially in place of 20,271Mstb. These were comparable with the field production data in all with ramifications, 98.64% accuracy.

The models were applied in an oil production test well which was test-run for only a month. The regional (field) data were projected to five years (generic) data as the models input data. The results showed an estimated cumulative oil production of 68,484.38 *Mstb*. The value was comparable to the tabulated tank values that summed up to a cumulative of 68,342.98 *Mstb*, with 99.98% accuracy.

Another application was done on a production test with projected production rates from 1996 to 2006. The results showed 99.75% accuracy 134,229M stb against 133,893.35Mstb.

III. RESULTS AND DISCUSSION

Table 3 shows the confirmed evaluation models for both projectile and parabolic dominated fluids flow.

TABLE III. CONFIRMED EVALUATION MODELS EQUATIONS

Eqns	Projectile Evaluation Model Equations	Remarks
49 71 73 74	$G_p = \frac{q_i}{2}[(t_2 - t_1) - (t_3 - t_0)]$ $G_p = q_i\left[t_2 - 0.5(t_1 + t_o) + \frac{(1 - e^{-bt_3})}{b}\right]$ $N_p = \frac{q_i}{2}[(t_2 - t_1) + (t_3 - t_0)]$ $N_p = q_i\left[t_2 - 0.5(t_1 + t_o) + \frac{(1 - e^{-bt_3})}{b}\right]$	Cumulative Gas & Oil Recovery, Fig 5 & Fig 6
72 75	$G = \frac{q_i}{2}[(t_2 - t_1) + (t_f - t_o)]$ $N = \frac{q_i}{2}[(t_1 - t_0) + (t_f - t_1)]$	Initial Gas/Oil in Place Fig 5 & Fig 6
60 67	$b = \frac{ln(q_i/q)}{t - t_i}$ & $q = q_i e^{-bt}$ $G_p = \frac{q_i}{b}(1 - e^{-bt})$ or $N_p = \frac{q_i}{b}(1 - e^{-bt})$	$\frac{dq}{dt} = -bq^n$ *For* $n = 1$
61	$b = \frac{q_i - q}{q\,t}$ & $q = \frac{q_i}{1+bt}$ $G_p = \frac{[q_i - q]}{b}$ or $N_p = \frac{[q_i - q]}{b}$	$\frac{dq}{dt} = -bq^n$ For $n = 2$
62	$b \approx \frac{1}{n}\left[\frac{q_i - q_1}{q_1\,t_1} + \frac{q_i - q_2}{q_2\,t_2} + . + \frac{q_i - q_n}{q_n\,t_n}\right]$ $q = \frac{q_i}{1+bt}$ & $G_p = \frac{[q_i - q]}{b}$ or $G_p = \frac{[q_i - q]}{b}$	$\frac{dq}{dt} = -bq^n$ When $n < 1$ or $n < 2$
Eqns	**Parabolic Evaluation Model Equations**	**Remarks**
	$G_p = \frac{q_i}{2}[(t_1 - t_0) + (t_2 - t_1)]$ $G_p = \frac{q_i}{2}[(t_1 - t_0) + (t_2 - t_1)]$ $G = \frac{q_i}{2}[t_f - t_0]$ OR $N = \frac{q_i}{2}[t_f - t_0]$	Cumulative & Initially fluids Estimations Fig 7/8

IV. DISCUSSION

The primary advantage of these models result is to identify the dominated flow trend of an oil or gas reservoir. This would enhance the prediction of the fluid production in a given period using the trend principal drive mechanism. At any stage of production (early, transient, transition or decline) it controls the flow performances. Two principal flow regimes projectile and parabolic dominated flows were delineated. The projectile dominated flow regime delineates conditions of saturated reservoirs and production wells located above the bubble point conditions. The parabolic dominated flow regime delineates parameters of the unsaturated energy drive system and below the bubble point conditions. The advantage in using generic data is mainly to enhance hydrocarbons production projected values. This makes it easy to predict future hydrocarbons production performances and take decision on the reservoir pressure management. The results showed high accuracy on the forecast. The percentage accuracy for gas fields ranged from 99.86% and above, while the percentage accuracy for oil ranged from 98.64% to 99.98%.

V. CONCLUSION

Mathematical models equations were successfully derived for studying reservoirs fluids depletion from the peak value to an economic value called abandonment. The models were equally used to predict or project future production performances of reservoirs and the projection trend could be used to estimate the reservoir initial fluids in place. The percentage accuracy obtained for gas fields ranged from 99.86% and above, while the percentage accuracy for oil ranged from 98.64% to 99.98%.

VI. RECOMMENDATIONS

- The models do not take care of pressure drawdown, so cumulative fluids production trend depend on pressure sustainability, hence it is recommended that the operator should use the curves trend in managing the reservoir pressure for high recovery economically.
- These evaluation model equations should be used as surveillance to monitor the production performances or decline trend. This would make it easy to know if the start or initial decline continues to the end of a production life in a reservoir.

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