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On the Field of a Moving Charge

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Abstract- An electro-magnetic field appearing in a laboratory due to moving charges has unusual properties. In particular, such a field of kinematical origin does not obey the wave equation with a non-relativistic velocity instead of light speed; so its movement resembles that of a rigid body. The present article deals with kinematical field in two extreme cases: non-relativistic and relativistic. A non-standard approach to the familiar laws deduced from a primary principle, on the one hand, and a new idea of the direct experimental verification for the well-known formula concerning the field of a uniformly moving point charge, on the other, are suggested.

Keywords- *Moving Charge, Relative Motion, Electro-Kinematics, Rowland-Eihenwald Effect, Lorentz Force and Biot-Savart Law, Ampere Force, Point Charge Field*

I. INTRODUCTION

What is the nature of science? The 19th century prominent British biologist-Darwinist Thomas Huxley gave such an answer: ‘It is simply common sense at its best – rigidly accurate in observation and merciless to fallacy in logic.’ Some provinces in the realm of Physics hitherto obey this determination, for instance classical electricity and magnetism. In the previous works [1-3] we presented the experimental results proving a natural existence of the “dragging” the magnetic field B by a moving at a speed v permanent magnet (magneto-kinematical Zajev-Dokuchajev effect), which induces in the laboratory electric field $\mathrm{E} = \mathrm{B} \times \mathrm{v}$ (in the SI units). It would be natural to expect the existence in the nature of a symmetric electro-kinematical phenomenon, conjugal to the Z-D effect. J. C. Maxwell dreamed and hoped to ‘at least verify our supposition that a moving electrified body is equivalent to an electric current’ [4, p.370, article 770]. Moreover, he suggested the real scheme of a relevant experimentation, but... ‘The unified view of electricity and magnetism which was then emerging from Maxwell’s work suggested that any moving charge ought to cause a magnetic field, but experimental proof was hard to come by’ [5, p. 241]. Not long after an American physicist Henry Rowland, working ‘*in the laboratory of the Berlin University through the kindness of Professor Helmholtz*’ [5 p. 242], first qualitatively showed an electro-kinematical effect in his experiments with the charged rotating disk. His article entitled ‘On the Magnetic Effect of Electric Convection’ was published in [6]. The scheme of the Rowland’s apparatus is given in [5, p. 242]. In chapter 9 entitled ‘*Maxwell`s equations for moving media*’ Panofsky and Phillips write: ‘*Hence the moving polarized dielectric will produce a magnetic field which is indistinguishable from that of a magnetized material. This effect has been demonstrated experimentally by Roentgen, Eihenwald, and others*’ [7, p. 165]. In the very beginning of the 20-th century A.A. Eihenwald (a Russian physicist with German roots) fulfilled the series of painstakingly organised experiments [8]. He worked so with the “Rowland’s current” as with the “Roentgen’s current” and managed to get one order higher precision than was obtained in the previous experiments. So Eihenwald quantitatively corroborated the existence of the magnetic field $\mathbf{B} = \frac{1}{c^2}(\mathbf{v} \times \mathbf{E})$ (too small in value because of the square light speed in the denominator), induced in the laboratory by moving at a speed $\boldsymbol{v}$ charged (or polarized) rigid bodies, so called convection currents. It would be fair to name this physical phenomenon the Rowland-Eihenwald effect (R-E effect) [9].

The principal scheme of an experimental arrangement, providing a way to reliably observe the R-E effect, is shown in figure 1. In a vacuum volume two thin aluminium disks make up capacitor plates. These disks are independently rotated by two electro-motors, variable in speed. A magneto-sensitive element is placed between disks. As a version, it may be placed in outer space if the case of vacuum volume is made of glass.

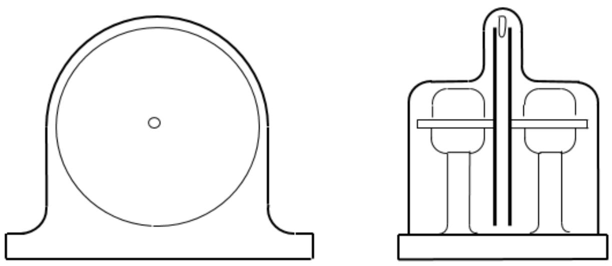

Figure 1. A modern apparatus for examination of the Rowland-Eihenwald effect

Let us appreciate quantitatively a possible kinematical magnetic field $\mathrm{B} = \frac{1}{c^2}(\mathrm{v} \times \mathrm{E})$ appearing due to R-A effect. Let the diameter of the rotating disk be $D = 0.5\, m$, the angle speed of rotation -- $\omega = 1000\, rev/sec$, and the strength of the electric field between disks $E = 1000\, kV/m$. So the linear velocity at the disk’s rim would be $1570\, m/sec$ and the appearing magnetic flux density of kinematical origin

$B = \frac{1}{c^2}\, vE = \frac{1570 \cdot 1000000}{9 \cdot 10^{16}} \simeq 1.75 \cdot 10^{-8}\, T$

$= 1.75 \cdot 10^{-4}\, Gauss$

This value is approximately ten times bigger than in Eihenwald's experiments, and hundred times as big as Rowland could reach. The natural magnetism on the Earth's surface is of the order of 0,5 $Gauss$, therefore the appropriate orientation of the apparatus relative to meridian is extremely important.

II. THE BIOT-SAVART LAW AND AMPERE LAW DEDUCED FROM THE ELECTRO-KINEMATICAL PRINCIPLE

In the tradition of contemporaneous education there are three types of approach to the Biot-Savart law: it is postulated [10, p 175, 11, p 215], is a consequence of the Ampere force between two currents [7, pp 123-5, 12, p 293-5], and it is theoretically deduced from the Maxwell equations [5, pp 223-5]. 'The fact that $\nabla \cdot \mathrm{A} \equiv 0$ opens the way to the proof of an important theorem, the result of which is known as Ampere's circular law' [13, p 244]. The mostly complete process starting from the differential equation for vector-potential A and using the vector analysis formalism is presented in [14, pp 495-6].

The general form of the Ampere-Maxwell law is $curl\ \mathrm{H} = \mathrm{j} + \partial \mathrm{D}/\partial t$ and for vacuum, where $\mathrm{B} = \mu_0 \mathrm{H}$, $\mathrm{D} = \varepsilon_0\, \mathrm{E}$, we have $curl\ \frac{1}{\mu_0}\mathrm{B} = \mathrm{j} + \varepsilon_0\, \partial \mathrm{E}/\partial t$ or $curl\ \mathrm{B} = \mu_0 \mathrm{j} + \varepsilon_0 \mu_0\, \partial \mathrm{E}/\partial t$. In the case of a stationary field

$$curl\ \mathrm{B} = curl\ curl\ \mathrm{A} = \mu_0 \mathrm{j}$$

This quantity may be rewritten using the well-known vector identity containing the Laplacian operator ∇^2: $curl\ curl\ \mathrm{A} \equiv grad\ div\ \mathrm{A} - \nabla^2 \mathrm{A}$. With the choice of the Coulomb gauge condition $div\ \mathrm{A} = 0$ we obtain the expression

$$\nabla^2 \mathrm{A} = -\mu_0 \mathrm{j}$$

which is a vector form of the Poisson equation. The general solution to this equation is

$$\mathrm{A}(\mathrm{r}) = \frac{\mu_0}{4\pi} \oint_V \frac{\mathrm{j}(\mathrm{r}')\mathrm{d}V'}{|\mathrm{r} - \mathrm{r}'|}$$

where integration is made over the volume V, containing the current distribution.

For a long thin wire the current density $\mathbf{j}(\mathbf{r})$ is zero everywhere except along the conductor. In the integrand it may be written through the current intensity and the direction vector of the wire at the point with position vector $\mathbf{r}'$: $\mathbf{j}(\mathbf{r}')\mathrm{d}V' = I\mathrm{d}\mathbf{l}'$. So the integral over the volume becomes an integral over the full length s of the closed conductor:

$$\mathrm{A}(\mathrm{r}) = \frac{\mu_0}{4\pi} \oint_s \frac{I\mathrm{d}l'}{|\mathrm{r} - \mathrm{r}'|}$$

Hence

$$\mathrm{B}(\mathrm{r}) = \frac{\mu_0 I}{4\pi}\, curl \oint_s \frac{\mathrm{d}l'}{|\mathrm{r} - \mathrm{r}'|} = \frac{\mu_0 I}{4\pi} \oint_s curl\ \frac{\mathrm{d}l'}{|\mathrm{r} - \mathrm{r}'|}$$

where the unprimed operator $curl$ means that the differentiation is with respect to the variables $x,\ y,\ z$. Another identity of the vector analysis gives us the following expression:

$$curl\,\frac{\mathrm{d}l'}{|\mathrm{r} - \mathrm{r}'|} \equiv \frac{1}{|\mathrm{r} - \mathrm{r}'|}\, curl\ \mathrm{d}l' + grad\ \frac{1}{|\mathrm{r} - \mathrm{r}'|} \times \mathrm{d}l'$$

Here the unprimed differentiation applied to the primed vector yields zero and so we obtain finally

$$\mathrm{B}(\mathrm{r}) = \frac{\mu_0 I}{4\pi} \oint_s grad\ \frac{1}{|\mathrm{r} - \mathrm{r}'|} \times \mathrm{d}l' = \frac{\mu_0 I}{4\pi} \oint_s \frac{\mathrm{d}l' \times (\mathrm{r} - \mathrm{r}')}{|\mathrm{r} - \mathrm{r}'|^3}$$

which is the Biot-Savart law.

All this tiresome procedure reminds us of the formal inference of the flux rule via the Stokes' theorem. 'Two situations in which e.m.f.s exist in closed circuits are considered in this book. First we discuss motional e.m.f.s, which are generated if part or all of a circuit moves in the laboratory frame when a steady magnetic field is present. Motional e.m.f.s can be calculated using the Lorentz force law, and for a closed circuit can be related to the change of magnetic flux through the moving circuit. This relationship obtained for motional e.m.f.s remains true for a second case we consider, that of induced e.m.f.s generated when a circuit is stationary but the magnetic field in which it sits is varying with time... ' [14, p. 213]. As it has been shown in [2], one could get rid of the long calculation in the case of moving magnetic source if the Z-D effect is taken into account, especially as the flux rule is useless in a case of the open circuit. In the present paper the possibility of a similar direct approach in the case of the Biot-Savart law for moving electric source would be proved.

Now we can deduce the two well-known laws of electro-magnetism on the base of the R-E effect. The above concept would be applied to deduce the well-known empirical laws matching electricity with magnetism. We start from the well-established experimental fact that a static electric field moves like a solid body synchronously with the charged source. First of all consider a point charge q moving relative to the laboratory reference frame along the X axis at a constant velocity $\mathbf{v}$ (figure 2). The magnetic vector due to the Rowland–Eihenwald effect in an arbitrary point A, having the radius vector $\boldsymbol{r}$, can be expressed (in SI units) through the electric vector as follows:

$$\mathrm{B_k} = \frac{1}{\mathrm{c}^2}(\mathrm{v} \times \mathrm{E}) = \frac{1}{\mathrm{c}^2}\left(\mathrm{v} \times \frac{\mathrm{qr}}{4\pi\varepsilon_0 \mathrm{r}^3}\right) = \frac{\mathrm{qv} \times \mathrm{r}}{\mathrm{c}^2 4\pi\varepsilon_0 \mathrm{r}^3}$$

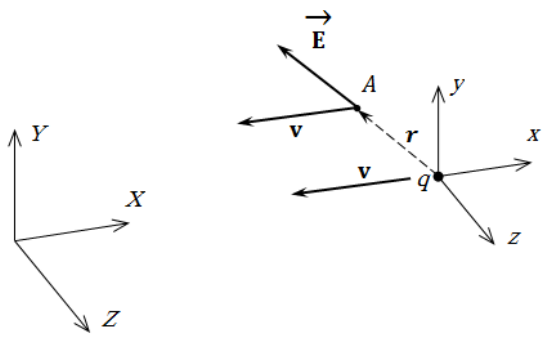

Figure 2. A point charge moving relative to the laboratory reference frame at a constant velocity

Let it exists a circled thin cord charged uniformly with a line density ρ (figure 3). It is moving on the set of pulleys in an anticlockwise direction so that every differentially small element $\boldsymbol{dl}$ has a linear velocity $\mathbf{v}$ parallel to former. This small enough element might be taken as a point charge ρdl moving with velocity $\mathbf{v} = v\boldsymbol{dl}/dl$. As above we can express the correspondent magnetic vector in an arbitrary point A:

$$dB_A = \frac{1}{c^2}\left(\frac{\rho dlvdl}{dl} \times \frac{r}{4\pi\varepsilon_0 r^3}\right) = \frac{v\rho}{4\pi\varepsilon_0 c^2}\left(\frac{dl\times r}{r^3}\right) = \frac{\mu_0 I}{4\pi}\left(\frac{dl\times r}{r^3}\right)$$

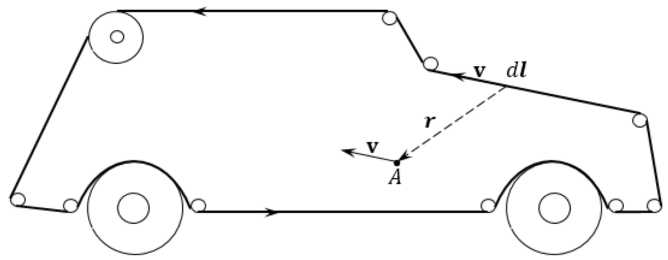

Figure 3. An example of a circled charged cord

where I is intensity of the convection current, and μ_0 is the absolute magnetic susceptibility. Thus, the law, which was intuitively developed from series of experimentation by two French physicists Jean-Baptiste Biot and Felix Savart about 1820, is derived now (two centuries later) from an underlying primary principle – kinematics of electricity.

Let $\boldsymbol{dl_1}$ and $\boldsymbol{dl_2}$ were two different linear elements of a charged cord (the same or sundry – does not matter). According to the Biot-Savart law a magnetic vector

$$dB_{12} = \frac{\mu_0 I_1}{4\pi}\left(\frac{dl_1\times r_{12}}{r_{12}^3}\right)$$

is present in the point $\boldsymbol{r_2}$, where the element $\boldsymbol{dl_2}$ is situated, and $\boldsymbol{r_{12}} = \boldsymbol{r_2} - \boldsymbol{r_1}$. Here the point charge $\rho_2 \boldsymbol{dl_2}$ moves with a velocity $\mathbf{v_2} = v_2\boldsymbol{dl_2}/dl_2$ and it is subject to the Lorentz force

$$dF_{12} = \rho_2 dl_2(\mathrm{v}_2 \times dB_{12}) = \frac{\mu_0 I_1}{4\pi}\left(\rho_2 dl_2 \cdot \frac{v_2}{dl_2}\frac{dl_2\times(dl_1\times r_{12})}{r_{12}^3}\right)$$

$$= \frac{\mu_0 I_1 \rho_2 v_2}{4\pi} \cdot \frac{dl_2\times(dl_1\times r_{12})}{r_{12}^3} = \frac{\mu_0}{4\pi}\frac{I_1 I_2}{r_{12}^3} \cdot \boldsymbol{dl_2} \times (\boldsymbol{dl_1} \times \boldsymbol{r_{12}})$$

From reasons of symmetry the natural expression

$$dF_{21} = \frac{\mu_0}{4\pi}\frac{I_2 I_1}{r_{21}^3} \cdot dl_1 \times (dl_2 \times r_{21})$$

follows for the force exerted by element $\boldsymbol{dl_2}$ on element $\boldsymbol{dl_1}$, where $\boldsymbol{r_{21}} = \boldsymbol{r_1} - \boldsymbol{r_2}$. This is the so called Ampere law for reciprocal magnetic interaction in vacuum between small linear conductors with currents. The total magnetic force between two closed circuits is obtained by integration.

It is interesting to note that the real wire with current represents two "cords": first one charged positively, which remains stationary, and second one charged negatively with the same charge density, which slides along with a velocity of several decimetres per hour (figure 4). If such a double cord is motoring on pulleys at any rate, this does not produces for good reason any change in the Ampere forces, operating between its portions. As is known, a cylindrical coil with current is subjected to a compression along the axis and to a stretching in the radial direction, and these stresses do not vary (naturally, associated centrifugal force taken into account) when this coil is rotating.

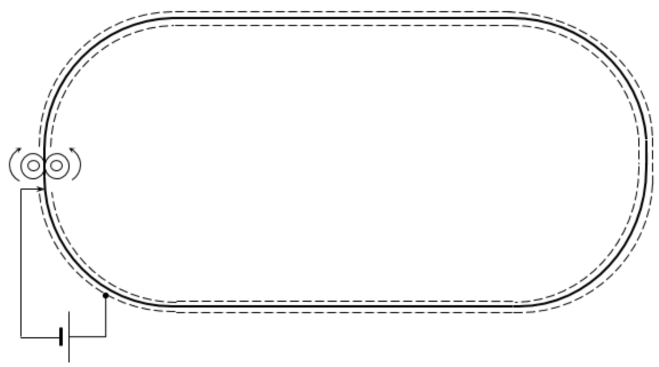

Figure 4. A double cord with opposite charges

III. THE CONTROVERSY ABOUT AMPERE LAW

Actually, Henry Ampere had enunciated his rule in the form [13, p 207]:

$$dF_{12} = \frac{\mu_0}{4\pi} I_1 I_2\{3(dl_1 \cdot r_{12})(dl_2 \cdot r_{12}) - 2\,(dl_1 \cdot dl_2)\}\frac{r_{12}}{r_{12}^3}$$

This formula gives the same result as an implementation of the modern formula

$$dF_{12} = \frac{\mu_0}{4\pi}\frac{I_1 I_2}{r_{12}^3} \cdot dl_2 \times (dl_1 \times r_{12})$$

when dF_{12} is integrated over all the elements of closed current. Tamm writes: "*within the range of examining closed steady currents the force of its interaction cannot be uniquely determined. Mathematically this circumstance reveals itself in the following: you may add any terms to the differential expression of the force, providing the full integral of these terms over close circuit being zero*" [15, pp 206-207]. But unlike the original Ampere enunciation the modern differential formula does not satisfy the third Newtonian law: equality of action and reaction. For example, if an element dl_1 is perpendicular to r_{12} and an element dl_2 is parallel to r_{12}, we have the force $dF_{12} \neq 0$, whereas $dF_{21} = 0$.

This circumstance for many years was the object of a heated discussion, see for example the paper by Victor Aleshinsky [16]. The application of a mathematical identity to the modern formula gives the following form:

$$dF_{12} = \frac{\mu_0}{4\pi}\frac{I_1 I_2}{r_{12}^3} dl_2 \times (dl_1 \times r_{12})$$

$$= \frac{\mu_0}{4\pi}\frac{I_1 I_2}{r_{12}^3}\{dl_1(dl_2 \cdot r_{12}) - r_{12}(dl_2 \cdot dl_1)\}.$$

Aleshinsky completes it with a new term

$$dF_{12} = \frac{\mu_0}{4\pi} \frac{I_1 I_2}{r_{12}^3} \{dl_1(dl_2.r_{12}) - r_{12}(dl_2.dl_1) + dl_2(dl_1.r_{12})\},$$

which make the differential (not only integral) force to obey the third Newtonian law. But the point is that it does not yet deliver electrodynamics from this "violation" of the law.

A point charge q being immobile in the laboratory has electrostatic field described by the Coulomb formula for electric intensity

$$\mathrm{E} = \frac{qr}{4\pi\varepsilon_0 r^3}.$$

If the charge moves with a constant velocity v, a magnetic field appears due to R-E effect, but hand in hand with this its electrostatic field is deformed. In Cartesian coordinates it is described [17, p 242] by the equation

$$\mathrm{E}(x, y, z) = \frac{q}{4\pi\varepsilon_0} \frac{\gamma \mathrm{r}}{(\gamma^2 x^2 + y^2 + z^2)^{\frac{3}{2}}},$$

where the charge is moving along the x-axis, $\gamma^2 = 1/(1 - v^2/c^2) = 1/(1-\beta^2)$, c is light speed. Using cylindrical symmetry the field may be presented as follows:

$$\mathrm{E}(\theta) = \frac{qr}{4\pi\varepsilon_0 r^3} \frac{1 - v^2/c^2}{\left(1 - \frac{v^2}{c^2}\sin^2\theta\right)^{\frac{3}{2}}}.$$

Here θ is angle between velocity v of the charge and radius-vector r which is directed to the test point [11, p 439]. These formulae are a simple consequence of the Liénard-Wiechert potentials. These potentials were enunciated several years before the birth of special relativity theory, which had corroborated their validity.

Mathematical expressions for $\mathrm{E}(x, y, z)$, for $\mathrm{E}(\theta)$ or both, you might find in many textbooks, but everywhere comments are either scarce or incorrect. Such a lack of due attention invalidates this remarkable field as a potential instrument for a cognition of natural enigmas.

IV. ELECTRIC FIELD OF A UNIFORMLY MOVING POINT CHARGE

For example, you could read: '*electric field of a uniformly moving point charge is flattened in the direction of movement*' [18, p 126] which is incomplete because the Coulomb field in motion is not only squeezed along but also is dilated across. And a little later: '*electric field of a fast moving charge at a fixed distance from it differs noticeably from zero in a narrow interval of angles near the equatorial plane, whereas the width of interval decreases as* $\sqrt{(1-\beta^2)}$ *with increase of velocity* v ' [18, p 127]. But why is it? -- No answer. Another instance: '*Because of the* $\sin^2\theta$ *in the denominator, the field of a fast-moving charge is flattened out like a pancake in the direction perpendicular to the motion. In the forward and backward directions* E *is reduced by a factor* $(1-\beta^2)$ *relative to the field of a charge at rest; in the perpendicular direction it is enhanced by a factor* γ' [11, p 439]. Here the former statement is wrong, and needed prove will be done in the next paragraphs.

Let us start with the expression for $\mathrm{E}(x, y, z)$ with a unit charge $q = 1$ from section 8 entitled "Relativity and electricity" in the book [17, p 242]. With a precision to a constant proportionality factor the field in the plane $z = 0$ is

$$\mathrm{E}(x, y) = \frac{\gamma \mathrm{r}}{(\gamma^2 x^2 + y^2)^{\frac{3}{2}}}.$$

The structure of any field comes to light due to the lines of equivalency. Consider an equation for the line where the vector $\mathrm{E}(x, y)$ has a constant absolute value ("level line"):

$$E(x, y) = \frac{\gamma\sqrt{x^2 + y^2}}{(\gamma^2 x^2 + y^2)^{\frac{3}{2}}} = a = const,$$

which may be rewritten in the form

$$F(x, y) = \frac{\sqrt{x^2 + y^2}}{(\gamma^2 x^2 + y^2)^{\frac{3}{2}}} - \frac{a}{\gamma} = 0.$$

The left side presents a function $F(x, y)$ of two variables x and y, depending on one another. So we have implicit function $x = x(y)$. A full differential of the function $F(x, y)$ is

$$dF(x, y) = \frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} dy \equiv 0,$$

where from the derivative

$$\frac{dx(y)}{dy} = -\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial x}}.$$

An appropriate calculation gives the expression

$$\frac{dx(y)}{dy} = \frac{y[(\gamma^2 - 3)x^2 - 2y^2]}{x[2\gamma^2 x^2 + (3\gamma^2 - 1)y^2]}.$$

For all $x \neq 0$ the denominator is positive and here it is possible to seek roots of the numerator. One of them is evident: $y_0 = 0$. In order to find all remaining roots from the equation $(\gamma^2 - 3)x^2 - 2y^2 = 0$ it is necessary to write off the term $x^2 = 2y^2/(\gamma^2 - 3)$ and substitute into the field equation for the level line $E(x, y) = a$. This provides the equation

$$\frac{\gamma(\gamma^2 - 3)}{y^2(\gamma^2 - 1)3^{3/2}} = a,$$

where from

$$y_{1,2} = \pm\sqrt{\frac{\gamma(\gamma^2 - 3)}{3^{3/2} a(\gamma^2 - 1)}} = \pm\sqrt{\frac{3\beta^2 - 2}{3^{3/2} a\beta^2 (1-\beta^2)^{1/2}}}$$

The equation of the level line $E(x, y) = a$ and the expression $x^2 = 2y^2/(\gamma^2 - 3)$ give us three corresponding values

$$x_0 = \sqrt{\frac{1}{a\gamma^2}} = \sqrt{\frac{1-\beta^2}{a}}, \quad x_{1,2} = \sqrt{\frac{2\gamma}{3^{3/2} a(\gamma^2 - 1)}} = \sqrt{\frac{2(1-\beta^2)^{1/2}}{3^{3/2} a\beta^2}}$$

for the front level line (arrows in the figure 5) and symmetrical negative values $(-x_0, -x_{1,2})$ for the rear level line. Three roots fuse together at the point $y_0 = 0$, if $\gamma^2 = 3$ ($\beta^2 = 2/3$ or $v/c \cong 0.8165$). Any two level lines $E(x, y) = a$ and $E(x, y) = b$ are homothetic with the dilatation ratio $= \sqrt{a/b}$, i.e. if $E(x, y) = a$, then $E(kx, ky) = b$.

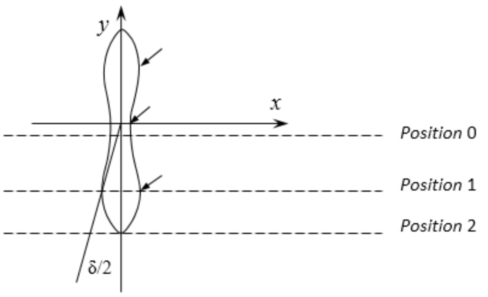

Figure 5. The "distorted" field of a moving point charge

Let the level of the line $E(x, y) = a$ in the figure 5 be threshold sensitivity for the laboratory electro-sensor device. As soon as

$$\tan\frac{\delta}{2} = \frac{x_{1,2}}{y_{1,2}} = \sqrt{\frac{2(1-\beta^2)}{3\beta^2-2}} = \sqrt{\frac{2}{\gamma^2-3}}$$

for the device, situated at a distance $b = |y_{1,2}|$ from the path of the moving charge (*position* 1 in the figure 5), the field is confined inside the angle δ. In this sense (and only in this!) the above citation from Landau [18, p 127] should be taken. The duration of the signal given by the electro-sensor 1 is $\Delta t = 2x_{1,2}/v$. The statement "A measure of the interval over which the fields are appreciable is evidently $\Delta t = b/\gamma v$" [10, p 560] is incorrect. In reality at the distance $b = \sqrt{\gamma/a}$ (*position* 2) the duration becomes zero. On the other hand, at the distance $b \cong x_0$ (*position* 0) the duration is $\Delta t \cong 1/\gamma v\sqrt{a}$. In the forward and backward directions **E** is reduced by a factor $(1-\beta^2)$ at the x-axis ***only***. Everywhere else it is less reduced or even enhanced. There are four "neutral" points with coordinates

$$x_n = \pm\sqrt{\frac{\gamma^{2/3}-1}{a(\gamma^2-1)}}, \; y_n = \pm\sqrt{\frac{\gamma^2-\gamma^{2/3}}{a(\gamma^2-1)}},$$

where electric fields both of the moving and immobile charges coincide.

In figure 6 some level lines (one quarter of each) are present for several values of velocity v. The arrows mark the points where the "pancake" has its maximal thickness. Moreover, the relation $x_{1,2}/x_0 \approx 0.6204\sqrt{\gamma}$ increases as γ increases so that any iso-surface $E(x, y, z) = a$ gets more and more akin to a doughnut, when v tends to c.

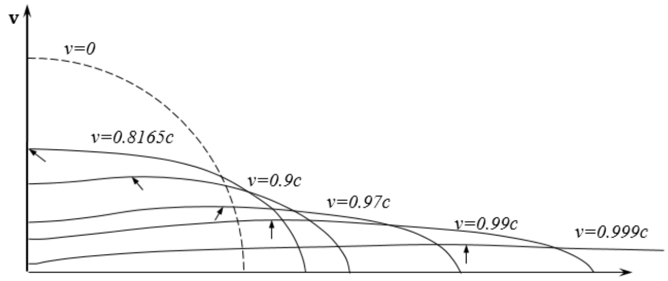

Figure 6. The fields of a point charge, moving with different velocities

V. About an "experimentum crucis"

The electric field of a fast-moving point charge proves to be much more nuanced than it appeared at first glance. Meanwhile, it has more important feature which could dot many pending 'i's. First of all recall breaching of the third law of mechanics: equality between action and counteraction. When two uniformly moving point charges have reciprocally perpendicular trajectories (figure 7), it might be a possible situation, where one charge experiences an action of the other but does not exert itself. Let both charges be electrons. The left charge is decelerated in the dilated field of the right one (figure 7(a)). The inverse situation is present in the figure 7(b): the upper charge is accelerated in the dilated field of the lower one.

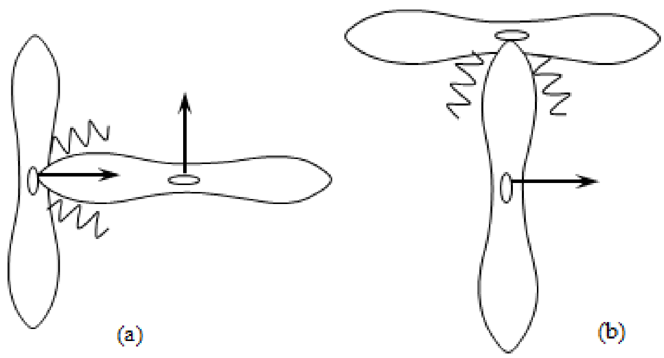

Figure 7. Electro-magnetic interaction between two moving point charges

Modern physics explains either case would be in violation of the third mechanical law in the following manner. A charge being accelerated or decelerated becomes the source of radiation and appearing photons carry away some definite part of momentum. The question arises: what experimental foundation has this problem? Here we are stumbling upon a quite sudden answer: ***nothing at all***. Odd as it may seem, the remarkable field of a uniformly moving point charge is the result of free imagination. Consider a method to establish whether this concept "holds water".

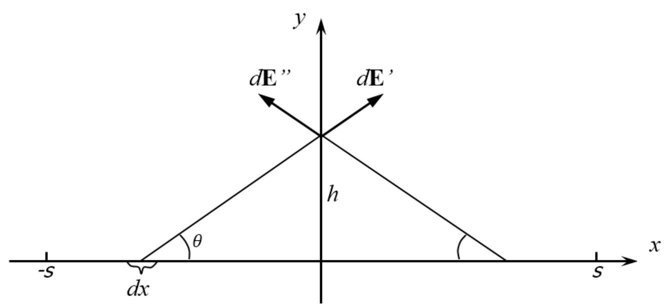

Figure 8. Experiment with a straight tube

Let a high-density electron beam exist in a rectilinear vacuum tube of the length $2s$ (x-axis in the figure 8). At a point $(0, h)$ element dx of the beam gives electric strength $d\mathrm{E}'$, and symmetrical element of the beam gives electric strength $d\mathrm{E}''$. The sum $d\mathrm{E}' + d\mathrm{E}'' = d\mathrm{E}$ is directed along the y-axis and has the absolute value

$$dE = \frac{2\sigma dx}{4\pi\varepsilon_0} \frac{(1-\beta^2)\sin\theta}{(x^2+h^2)(1-\beta^2\sin^2\theta)^{\frac{3}{2}}} = \frac{\sigma(1-\beta^2)dx}{2\pi\varepsilon_0[x^2+(1-\beta^2)h^2]^{\frac{3}{2}}},$$

using the linear charge density σ, and taking into account, that $\sin\theta = h/\sqrt{x^2+h^2}$. Full electric strength is obtained by integration over all beam:

$$E = \int_0^s dE = \frac{\sigma(1-\beta^2)}{2\pi\varepsilon_0} \int_0^s \frac{dx}{[x^2+(1-\beta^2)h^2]^{\frac{3}{2}}} = \frac{\sigma}{2\pi\varepsilon_0} \cdot \frac{s}{h[s^2+(1-\beta^2)h^2]^{\frac{1}{2}}}.$$

Substituting the linear charge density through the speed of electrons in the beam and the current strength $I = \sigma v$, we have

$$E(h, s, \beta, I) = \frac{Is}{2\pi\varepsilon_0 vh\sqrt{s^2+(1-\beta^2)h^2}} = \frac{Is}{2\pi\varepsilon_0 hc\beta\sqrt{s^2+(1-\beta^2)h^2}}.$$

If the electric field of a moving point charge remains Coulomb's irrespective of velocity, the dependence on β would be another:

$$E_0(\beta) = \frac{Is}{2\pi\varepsilon_0\beta ch\sqrt{s^2+h^2}}.$$

The first idea of this procedure has been presented in a report to the local Siberian conference [19, pp 145-7). Unfortunately, both these functions are monotone and a tangible difference between them appears at a big distance h from the beam, where field's strength becomes rather small.

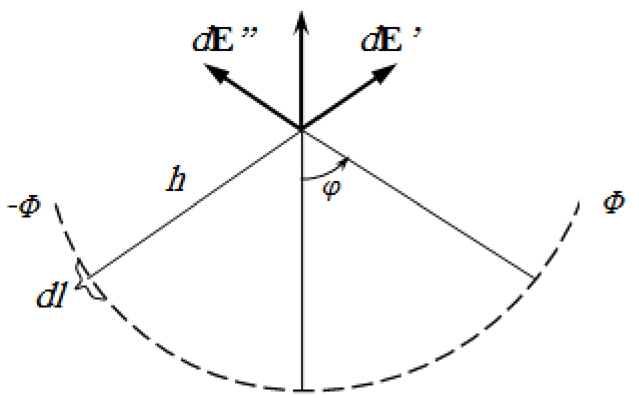

Figure 9. Experiment with an arc tube

Consider a new version of the experiment, which scheme is present in figure 9. The vacuum tube is a circular arc of radius h, the electron beam is bent in an appropriate uniform magnetic field, and the electric sensor is placed at the centre of circle. In this case the angle $\theta = \pi/2$ at every point in the trajectory and

$$dE = \frac{\sigma dl}{2\pi\varepsilon_0} \frac{(1-\beta^2)\cos\varphi}{h^2(1-\beta^2)^{\frac{3}{2}}} = \frac{\sigma\cos\varphi d\varphi}{2\pi\varepsilon_0 h\sqrt{(1-\beta^2)}}.$$

The field strength in the centre is

$$E(h, \Phi, \beta, I) = \int_0^\Phi dE = \int_0^\Phi \frac{\sigma d\varphi}{2\pi\varepsilon_0 h\sqrt{(1-\beta^2)}} = \frac{\sigma\sin\Phi}{2\pi\varepsilon_0 h\sqrt{(1-\beta^2)}}$$

$$= \frac{I\sin\Phi}{2\pi\varepsilon_0 hc\beta\sqrt{(1-\beta^2)}}.$$

This function offers some advantage over the previous one. First, its values are several times bigger at the same parameters. The key feature is always non-monotone type of dependence on speed of electrons in the beam. The U-shaped dependence with a minimum at $\beta = 1/\sqrt{2}$ furnishes an opportunity to reliably differentiate relativistic field from the Coulomb's one and other supposed options. It should be noted that the trajectory's curvature is conjugated with a nonzero acceleration and so "synchrotron" radiation would be inevitable, but this circumstance does not rescind anyhow the Liénard-Wiechert formalism.

VI. Conclusion

Unlike the involved mathematic used in modern textbooks, the electro-kinematical approach gives us a straight and short way to both the Biot-Savart and Ampere laws. Furthermore, this way is always valid irrespective of what gauge choice is made, because these laws are not electro-dynamical by character. Quite the reverse, they are basically electro-kinematical in their very nature. The main implication of the above analysis is that there exists a deep symmetry between electricity and magnetism not only in the dynamics but also in the kinematics, namely in the case of moving stationary fields. The Ampere force of electro-kinematical origin is the adjacent counterpart for the magneto-kinematical Lorentz force. Nature likes beauty.

For dynamical fields it is impossible to state a cause-effect relation between electricity and magnetism: electric and magnetic vectors in the electromagnetic wave are equiphase. For the field kinematics, on the contrary, the Ampere force, involved in the case of electro-kinematics, is an effect of the electric cause, and vice versa, the inverse Lorentz force, involved in the case of magneto-kinematics, is an effect of the magnetic cause. In the sequel, there are no electro-magneto-kinematical waves in the nature.

It is very important to examine experimentally the character of warping the field of a fast moving point charge. The simple scheme of appropriate experiments suggested above provides feasibility of the idea with use of a rather simple particle accelerator. For realization it is not necessary to build a "supercollider". The energy less than 1 Mev for electrons in the beam would be quite sufficient.

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