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Finding Optimum Subreflector Dimensions in Cassegrain Antenna Using Newton-Raphson Method

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***Abstract*-** In this paper, in order to find the optimal dimensions of the subreflector for minimizing the effects of aperture obstruction, the complex and nonlinear equation related to the design of the Cassegrain antenna and feed antenna, which is a conical corrugated horn antenna, is presented as a single-variable equation versus '*A*', one of the parameters of the hyperbolic reflector surface. Then this equation is solved using the Newton-Raphson numerical method with high accuracy and high speed. At the end of the paper, a design example was investigated to reveal the above method and the optimal dimensions of the Cassegrain antenna structure were calculated using this method.

***Keywords*-** *Conical Corrugated Horn Antenna, Cassegrain Antenna, Optimal Aperture Efficiency, Solving Single-Variable Nonlinear Equation, Newton-Raphson Method*

I. INTRODUCTION

Cassegrain antennas are dual-reflector antennas that have many benefits in satellite and terrestrial communications. Usually, in these antennas, conical corrugated horn antennas are used as feed antenna to create symmetric electric field patterns on E and H planes, whereas phase centers are also matched on these planes. There are two parabolic and hyperbolic reflectors in the Cassegrain antennas, and the feed antenna phase center is located on one of the focal points of the hyperbolic reflector, and the other focal point of the hyperbolic surface corresponds to the focal point of the parabolic surface. In this case, the waves that come out of the horn antenna's phase center after reflection from hyperbolic surface are transmitted toward the parabolic reflector and after reflection from the parabolic reflector propagate parallel to the antenna axis, and so this antenna has a very high gain.

The main issue in designing this antenna is to calculate the dimensions of the Cassegrain antenna and feed antenna, so that the antenna's radiation efficiency be optimal. In [1], the method of obtaining a relationship in terms of dimensions and specifications of the Cassegrain antenna and the conical corrugated horn antenna is investigated so that the efficiency of the aperture is maximal. This equation is non-linear and complex and can be solved with various numerical methods. The methods of solving this equation, which have been used so far, have a low convergence rate.

In this paper, we want to solve this equation using the Newton-Raphson method with fast convergence rate and high resolution. As far as the author knows, this equation so far has not been resolved using this method. In the second part of the paper, we first describe the Newton-Raphson method for solving non-linear equations, and the iteration relation is obtained for solving the problem of designing antenna dimensions. Then, in the third section, a design example is presented. This example is solved using Newton-Raphson's method with high speed and accuracy, and the results are presented.

II. THE NEWTON-RAPHSON METHOD FOR SOLVING THE DESIGN EQUATION OF CASSEGRAIN AND FEED ANTENNA

As shown in Fig.1, The Newton-Raphson method is used to solve the nonlinear equation $f(x)=0$ [2]. In this method, starting with an initial point x_0, a tangent line is drawn first on the curve at the point $(x_0,\ f(x_0))$. The intersection of this line with the x-axis, the point x_1, is considered as the next estimate of the root of the function. This process continues to the extent that the difference between $x_n - x_{n-1}$ reaches the required amount ε. We find the following relation to get the iteration relation of the Newton-Raphson's method,

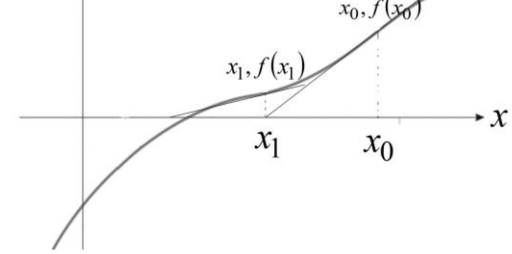

Figure 1. Graphical description of solving a single-valued equation by Newton-Raphson method.

$$f'(x_0)=\frac{f(x_0)}{x_0-x_1}\Rightarrow x_1=x_0-\frac{f(x_0)}{f'(x_0)} \quad (1)$$

Therefore, in the ith repetition step, we can write the following equation for calculating a new estimate of the root

$$x_i=x_{i-1}-\frac{f(x_{i-1})}{f'(x_{i-1})} \quad (2)$$

It is shown that in this method, the true error in the ith repetition step is related to the true error in (i-1)th step as follows

$$E_i=-\frac{f''(x_{i-1})}{2f'(x_{i-1})}E_{i-1} \quad (3)$$

that E_i is the difference between x_i and the real answer of the problem. Therefore, in this method by choosing an appropriate initial point in a sequence that function derivative does not become zero, depending on the function form around the root, the function root can be found with high speed and precision. The design equation for Cassegrain antenna to minimize the amount of aperture obstruction is as follows [1]

$$f(A)=4F\tan\left(\frac{\tan^{-1}\left(\frac{a_f}{(2Ae-L_p)}\right)}{2}\right)-2\frac{A(e^2-1)\sin(\psi_0)}{(1+e\cos(\psi_0))} \quad (4)$$

in this relation, '*A*' is the surface parameter of the hyperbolic reflector in the Cassegrain antenna, '*e*' is the eccentricity factor of the Cassegrain antenna and the hyperbolic reflector, and is related with magnification factor of the antenna, *M*, as follows

$$e=\frac{M+1}{M-1} \quad (5)$$

In (4), ψ_0 is the subtended angle of the parabolic reflector, and it is obtained in terms of the ratio of parabolic reflector diameter, *D*, to *F*, which is the focal length of the parabolic reflector, as

$$\psi_0=2\tan^{-1}\left(\frac{D}{4F}\right) \quad (6)$$

a_f is the radius of the feed antenna openings considering the depth of the slot on the antenna aperture plane, and equals with, $a_f=a+d$, that '*a*' is the radius of the outlet of the horn antenna, regardless of the depth of the slot, and '*d*' is the depth of the slot located at the outlet of the feed antenna and may be obtained as following,

$$d=\frac{\lambda}{4}\exp\left(\frac{1}{2.5ka}\right) \quad (7)$$

where λ is the free space wavelength and $k=2\pi/\lambda$. In relation (4), L_p is the displacement of the phase center of the feed antenna relative to the antenna aperture plane. It depends on a parameter called slant ratio of the horn, S, and the bending radius of the cone, R, and is determined according to Table 1. [3]. '*S*' is obtained by,

$$S=\frac{a^2}{2\lambda R} \quad (8)$$

that '*a*' is the radius of the feed antenna output, regardless of the depth of the output slot, and *R* is the bending radius of the cone.

In the design of the antenna dimensions, Fig. 2 is used [3], which gives the beam of the electric field of the conical corrugated horn antenna in terms of (9) for different values of *S*,

$$2\pi a\sin(\theta_0)/\lambda \quad (9)$$

In (9), θ_0 is the subreflector subtended angle and is obtained from,

$$\theta_0=2\tan^{-1}\left(\frac{D}{4F_e}\right) \quad (10)$$

Where F_e is the equivalent focal length of the Cassegrain antenna and equals *M* times *F*. When using Fig. 2, we assume that the relative electric field intensity at antenna edges is -11db = 0.2818 [4] which corresponds to the case of optimum radiation efficiency.

TABLE I. VARIATIONS OF THE RATIO OF L_p TO *R* VERSUS CHANGES IN *S* PARAMETER.

S	$\frac{L_p}{R}$	S	$\frac{L_p}{R}$
0	0	0.36	0.386
0.04	0.005	0.4	0.464
0.08	0.02	0.44	0.542
0.12	0.045	0.48	0.614
0.16	0.08	0.52	0.673
0.2	0.124	0.56	0.718
0.24	0.178	0.6	0.753
0.28	0.24	0.64	0.783
0.32	0.31	0.68	0.811

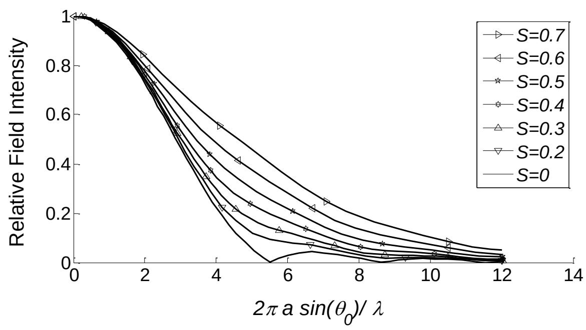

Figure 2. Variation of the relative electric field intensity of the conical corrugated horn antenna in terms of variations of the slant ratio factor.

With regard to the above, if we consider the proper values for the parameter *M*, the magnification of the Cassegrain antenna and *S*, the slant ratio of the horn antenna, in (4) all the parameters will be determined, except for the parameter *A*. We use the Newton-Raphson method for solving (4). The repetition equation in accordance with (2) is considered as follows

$$A_{i+1} = A_i - \frac{f(A_i)}{f'(A_i)} \tag{11}$$

$f(A_i)$ is obtained from (4) and $f'(A_i)$ may be calculated from

$$f'(A_i) = 4F \frac{\frac{1}{2}\frac{\frac{-2ea_f}{(2A_ie - L_p)^2}}{1+\left(\frac{a_f}{(2A_ie-L_p)}\right)^2}}{\cos^2\left(\frac{\tan^{-1}\left(\frac{a_f}{(2A_ie-L_p)}\right)}{2}\right)} - 2\frac{(e^2-1)\sin(\psi_0)}{(1+e\cos(\psi_0))} \tag{12}$$

After some algebraic simplifications we have,

$$f'(A_i) = \frac{-4Fea_f}{\left((2A_ie-L_p)^2 + a_f^2\right)\cos^2\left(\frac{\tan^{-1}\left(\frac{a_f}{(2A_ie-L_p)}\right)}{2}\right)} - 2\frac{(e^2-1)\sin(\psi_0)}{(1+e\cos(\psi_0))} \tag{13}$$

Now, using an example, the initial design steps of the Cassegrain antenna and the feed antenna and the method for obtaining parameter '*A*' are brightened.

III. Design example which describes the steps to find the optimal dimensions of the subreflector

Suppose we want to design a Cassegrain antenna and a conical corrugated horn antenna that receives satellite waves at center frequency of 15 GHz, and the parabolic reflector diameter is calculated equal to 8 (m), based on the link budget and the received signal strength. Suppose that the antenna magnification is optionally chosen as 5 and the parameter *S*, the slant factor of the feed antenna, is 0.2. In this case, considering the conditions for creating the smallest obstruction, we want to find the dimensions of the hyperbolic reflector by solving (4) with Newton-Raphson numerical method. Initially, according to (5), *e* = 1.5, and if the ratio *F* / *D* = 0.5 is considered, then from (6) $\psi_0 = 53.1^\circ$, and given that $F_e / D = MF / D = 2.5$, from(10), $\theta_0 = 11.42^\circ$, and therefore, with consideration of the Fig.1 diagram , For the case where the relative field strength at the antenna edges is -11 dB = 0.2818 and for *S* = 0.2, we have:

$$2\pi a \sin(\theta_0)/\lambda = 3.85 \rightarrow a = \frac{3.85\lambda}{2\pi\sin(\theta_0)} \tag{14}$$

$\lambda = c/f$ so the output radius of horn antenna becomes

$$a = \frac{3.85\times 3\times 10^8}{2\pi\sin(11.42^\circ)\times 15\times 10^9} = 6.2\ (cm) \tag{15}$$

Then from (7)

$$d = \frac{\lambda}{4}\exp\left(\frac{1}{2.5ka}\right) = \frac{3\times 10^8}{4\times 15\times 10^9}\exp\left(\frac{3\times 10^8}{2.5\times 2\pi\times 15\times 10^9\times 6.2\times 10^{-2}}\right) = 5.1\ (mm) \tag{16}$$

So $a_f = a + d = 6.2 + 0.51 = 6.71\,(cm)$

To calculate L_P, knowing *R* and *S* we can use Table 1. From (8),

$$R = \frac{a^2}{2\lambda S} = \frac{(6.2\times 10^{-2})^2\times 15\times 10^9}{2\times 3\times 10^8\times 0.2} = 48.05\,(cm) \tag{17}$$

From Table 1, for *S*=0.2, $\frac{L_p}{R} = 0.124$ and so, $L_p = 0.124\times 48.05 = 5.96\ (cm)$.

Now all the parameters in (4) are clear and this equation can be solved. To solve this equation by the Newton-Raphson method, it is necessary to select an initial value that, in accordance with equation (3), the derivative of the function in the neighborhood of that point does not get close to zero. To do this, you need to draw a function to examine its behavior around the root. Given the physics of the problem, knowing that the feed phase center locates in the space between the parabolic reflector and the hyperbolic reflector and also since the focal length of the parabolic reflector is 4 meters, so the maximum of *2C* can be four meters, and since the ratio *C* / *A* = *e* is greater than 1, So we can consider the range of variations of *A* in the range between 0 and 2m. With reference to Fig. 3, it is seen that the root of equation (4) is located around 0.5 and the root can be obtained by selecting the appropriate initial point. Here we start with $A_0 = 0.3$ to begin the root finding process and a computer program using MATLAB [5] is written. The values of A_i and $f(A_i)$ for the sequential iteration of the numerical method are given in Table 2. Here, the accuracy of the calculation is assumed to be 0.005, that is, when the difference, $|A_i - A_{i-1}| < 0.005$ satisfies, the root finding process stops.

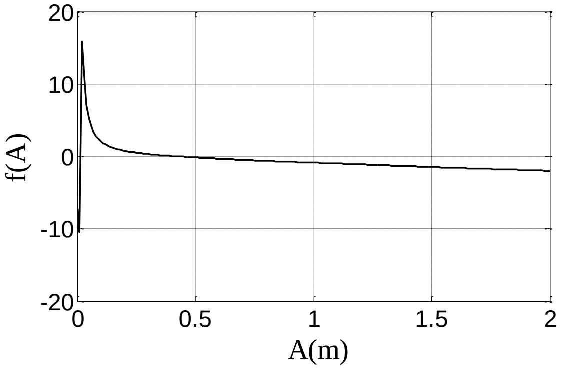

Figure 3. Ddiagram of equation (4).

TABLE II. ITERATIONS OF THE NUMERICAL METHOD AND THE VALUES OF THE FUNCTION AND THE ESTIMATED ROOT FOR EACH OF THE REPETITIONS.

iteration	A_i	$f(A_i)$
0	0.2 (m)	0.7792
1	0.3198 (m)	0.2593
2	0.4053 (m)	0.0375
3	0.4220 (m)	0.00082
4	0.4223 (m)	0.00038

As can be seen, after four iterations, this method converges and the root of equation (4) is calculated as 0.4223, and for this answer, the value of equation (4) decreases to about 0.00038. In this problem we arrive at the root of the equation at high speed and the advantages of the Newton-Raphson method are used to compute the parameter '*A*' for the hyperbolic surface. After calculating '*A*', the '*C*' parameter for the hyperbolic surface can also be obtained according to the following equation and the stages of designing the parabolic, hyperbolic surfaces and feed antenna are completed.

$$e = C / A \rightarrow C = eA = 1.5 \times 0.4223 = 0.6335 \quad (18)$$

IV. CONCLUSION

In this paper, the Newton-Raphson numerical method is used to solve the characteristic equation that determines the dimensions of the hyperbolic reflector and finds the root of this equation with high accuracy and high convergence rate. Initially, a description of finding the root of a nonlinear equation by the Newton-Raphson numerical method is presented. Then the characteristic equation of the antenna and its various variables, are presented and the initial design method of these variables is mentioned. In the following, a design example is investigated that further illustrates the method of calculating various variables in the characteristic equation and the method of calculating the root of the characteristic equation.

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