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Control of Vehicle Active Suspensions with Actuator Delay via Distributed Backstepping

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Abstract- In this study we examine the actuator time delay problem for a full vehicle active suspension system using the theory of backstepping control design. It is aimed to improve the ride comfort of passengers in case of actuator time delay, which may arise in active suspension systems because of information processing, sensors or some mechanical reasons. Particularly designing the controller without taking into account the actuator time delay may degrade the performance of the controller or even destabilize the closed loop suspension system. Therefore, we design a backstepping controller that takes into account the actuator time delay by combining a first order hyperbolic partial differential equation (PDE) with the suspension system. The numerical results confirm the success of the controller in improving ride comfort of the passengers while assuring the stability of the system.

Keywords- *Actuator Time Delay, Vehicle Active Suspension System, Distributed Backstepping Control*

I. INTRODUCTION

Suspension systems have an important role on comfort and safety of the vehicle ride and they are generally classified as passive, semi-active and active systems. Passive ones are composed of spring and damper elements whereas semi-active ones include variable damping elements such as electrorheological [1] and magnetorheological [2] dampers. In active suspension systems hydraulic, pneumatic or linear electric motors can be placed generally parallel to the suspension elements. Active suspensions provide promising performance for suppression of vehicle body vibrations if compared with passive and semi-active suspension systems. This is why this research area has remained attractive for many years and various control strategies such as PID [3], fuzzy logic [4], [5], H_∞ [6], sliding mode [7], [8], fuzzy sliding mode [9], [10], backstepping control [11], linear parameter varying control [12], [13], model predictive control [14], [15] have been proposed for the control of active suspension systems.

Though there are many studies concerning active suspension systems as mentioned previously, most of them neglect the time delay during the controller design. However in practice it is not possible to calculate and apply the needed control action to the system without any time delay. In active suspension systems the time delays exist mainly because of the actuator dynamics [16]. It is demonstrated in [17] that force tracking performance of hydraulic servo systems is limited. Therefore in reality, effect of time delay should be taken into account. If not, the performance of the controlled system may degrade or even cause instability of the system. Various approaches have been used in literature for the control of active suspensions with actuator delay. In [18] constrained optimization was used to calculate state feedback gains along with a scheme for stability chart strategy for quarter active suspension system. H_∞ control design have also been proposed for vehicle active suspension system with actuator delay in [19] and [20]. In [16], a parameter-dependent controller is designed for the same phenomena by solving a finite number of matrix inequality conditions for the design of controller.

In [21], to obtain a controller for a desired level for linear time invariant (LTI) systems, a boundary backstepping controller is designed by combining a first order hyperbolic partial differential equation (PDE) with LTI system. Mostly, boundary control is used for distributed systems by using backstepping design [22]. In [23] it is shown that this methodology can also be used for the delay systems by solving a coupled LTI-PDE system. With the same methodology of the designing backstepping controller, if a target stable system is chosen for the partial differential system, one can define a controller for the investigated delay system [23] by using the transformation between the original and the target systems. At the end a controller can be derived as smith predictor. In [24] many different types of Smith Predictor controllers are given that use different PDE systems for LTIs by using the same methodology. The stability analyses of the systems were also given in [24].

The distributed backstepping control(DBC) approach presented in [21] gives the ability to handle time delay systems in a more systematic way if compared with existing control methods such as H_∞ [19], [20] or linear matrix inequality based control [16]. Therefore, as the main contribution of this study, we have used that distributed backstepping approach presented in [21] for the vibration suppression of an active suspension system where actuator time delay exists. To the best knowledge of the authors the controller used in this study have not been yet used for active suspension control in literature.

With the numerical results presented in this study, it was shown that the designed controller improved vehicle ride comfort while assuring stability of the closed loop active suspension system in the presence of actuator time delay. Rest of the paper is organized as follows, the effects of actuator time delay on the operation performance of the active suspension system is first introduced in Section II. Afterwards, in Section III the distributed backstepping controller design is presented. Then, numerical results for the passive and active suspension systems are presented in Section IV and finally, conclusions are drawn in last section.

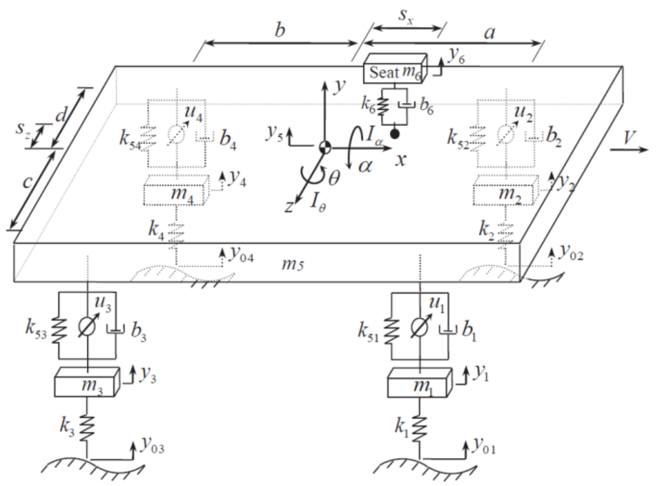


Figure 1. Full Vehicle active suspension system

II. PROBLEM FORMULATION

Full vehicle active suspension system model with a seat, presented in Figure 1, is used in this study. Mathematical model of the system is given in (1)-(8), where linearization for small angles of pitch and roll motions are carried out. The model has eight degrees of freedom which are bounce y_5, pitch θ and roll α motions of the vehicle body and displacement of the wheels $y_i (i=1,2,3,4)$ that are all in vertical directions. Here, $y_{0i} (i=1,2,3,4)$ is the road surface input representing the road surface unevenness. The $m_i (i=1,2,3,4)$ represent the mass of the wheel-axle assembly and m_5 is the mass of the vehicle main body, and m_6 is the mass of the seat, $k_i (i=1,2,3,4)$ is the stiffness constant of the tire; similarly $k_{5i} (i=1,2,3,4)$ and $b_i (i=1,2,3,4)$ stand for the stiffness and damping constant of the suspension spring and damper, respectively; $u_i(t-D)(i=1,2,3,4)$ is the control signal with time delay D. Numerical values of the vehicle parameters are given in Appendix. The road input applied to the vehicle suspension is shown in Figure 2a. The vehicle travels over that road profile with a constant velocity of $20\,m/s$. The road surface input effect the rear tire, $\delta = \dfrac{a+b}{V}$ s later than the front ones, because of the distance $a+b$ between the front and the rear axles.

$$\begin{aligned}
& I\ddot{\alpha} - k_{51}\cdot c\left(y_5 + a\cdot\theta - c\cdot\alpha - y_1\right) + k_{52}\cdot d\left(y_5 + a\cdot\theta + d\cdot\alpha - y_2\right) \\
& -k_{53}\cdot c\left(y_5 - b\cdot\theta - c\cdot\alpha - y_3\right) + k_{54}\cdot d\left(y_5 - b\cdot\theta + d\cdot\alpha - y_4\right) \\
& -k_6\cdot d\left(y_6 - y_5 - a\cdot\theta - d\cdot\alpha\right) - b_1\cdot c\left(\dot{y}_5 + a\cdot\dot{\theta} - c\cdot\dot{\alpha} - \dot{y}_1\right) \\
& +b_2\cdot d\left(\dot{y}_5 + a\cdot\dot{\theta} + d\cdot\dot{\alpha} - \dot{y}_2\right) - b_3\cdot c\left(\dot{y}_5 - b\cdot\dot{\theta} - c\cdot\dot{\alpha} - \dot{y}_3\right) \\
& +b_4\cdot d\left(\dot{y}_5 - b\cdot\dot{\theta} + d\cdot\dot{\alpha} - \dot{y}_4\right) - b_6\cdot d\left(\dot{y}_6 - \dot{y}_5 - a\cdot\dot{\theta} - d\cdot\dot{\alpha}\right) \\
& = -cu_1 + du_2 - cu_3 + du_4
\end{aligned} \tag{1}$$

$$\begin{aligned}
& I\ddot{\theta} + k_{51}\cdot a\left(y_5 + a\cdot\theta - c\cdot\alpha - y_1\right) + k_{52}\cdot a\left(y_5 + a\cdot\theta + d\cdot\alpha - y_2\right) \\
& -k_{53}\cdot b\left(y_5 - b\cdot\theta - c\cdot\alpha - y_3\right) - k_{54}\cdot b\left(y_5 - b\cdot\theta + d\cdot\alpha - y_4\right) \\
& +k_6\cdot a\left(y_6 - y_5 - a\cdot\theta - d\cdot\alpha\right) + b_1\cdot a\left(\dot{y}_5 + a\cdot\dot{\theta} - c\cdot\dot{\alpha} - \dot{y}_1\right) \\
& +b_2\cdot a\left(\dot{y}_5 + a\cdot\dot{\theta} + d\cdot\dot{\alpha} - \dot{y}_2\right) - b_3\cdot b\left(\dot{y}_5 - b\cdot\dot{\theta} - c\cdot\dot{\alpha} - \dot{y}_3\right) \\
& -b_4\cdot b\left(\dot{y}_5 - b\cdot\dot{\theta} + d\cdot\dot{\alpha} - \dot{y}_4\right) - b_6\cdot a\left(\dot{y}_6 - \dot{y}_5 - a\cdot\dot{\theta} - d\cdot\dot{\alpha}\right) \\
& = au_1 + au_2 - bu_3 - bu_4
\end{aligned} \tag{2}$$

$$\begin{aligned}
& m_1\cdot\ddot{y}_1 + k_1\left(y_1 - y_{01}\right) - k_{51}\left(y_5 + a\cdot\theta - c\cdot\alpha - y_1\right) \\
& -b_1\left(\dot{y}_5 + a\cdot\dot{\theta} - c\cdot\dot{\alpha} - \dot{y}_1\right) = -u_1
\end{aligned} \tag{3}$$

$$\begin{aligned}
& m_2\cdot\ddot{y}_2 + k_2\left(y_2 - y_{02}\right) - k_{52}\left(y_5 + a\cdot\theta + d\cdot\alpha - y_2\right) \\
& -b_2\left(\dot{y}_5 + a\cdot\dot{\theta} + d\cdot\dot{\alpha} - \dot{y}_2\right) = -u_2
\end{aligned} \tag{3}$$

$$\begin{aligned}
& m_3\cdot\ddot{y}_3 + k_3\left(y_3 - y_{03}\right) - k_{53}\left(y_5 - b\cdot\theta - c\cdot\alpha - y_3\right) \\
& -b_3\left(\dot{y}_5 - b\cdot\dot{\theta} - c\cdot\dot{\alpha} - \dot{y}_3\right) = -u_3
\end{aligned} \tag{3}$$

$$\begin{aligned}
& m_4\cdot\ddot{y}_4 + k_4\left(y_4 - y_{04}\right) - k_{54}\left(y_5 - b\cdot\theta + d\cdot\alpha - y_4\right) \\
& -b_4\left(\dot{y}_5 - b\cdot\dot{\theta} + d\cdot\dot{\alpha} - \dot{y}_4\right) = -u_4
\end{aligned} \tag{3}$$

$$\begin{aligned}
& m_5\cdot\ddot{y}_5 + k_{51}\left(y_5 + a\cdot\theta - c\cdot\alpha - y_1\right) + k_{52}\left(y_5 + a\cdot\theta + d\cdot\alpha - y_2\right) \\
& +k_{53}\left(y_5 - b\cdot\theta - c\cdot\alpha - y_3\right) + k_{54}\left(y_5 - b\cdot\theta + d\cdot\alpha - y_4\right) \\
& +b_1\left(\dot{y}_5 + a\cdot\dot{\theta} - c\cdot\dot{\alpha} - \dot{y}_1\right) - k_6\left(y_6 - a\cdot\theta - y_5 - d\cdot\alpha\right) \\
& +b_2\left(\dot{y}_5 + a\cdot\dot{\theta} + d\cdot\dot{\alpha} - \dot{y}_2\right) + b_3\left(\dot{y}_5 - b\cdot\dot{\theta} - c\cdot\dot{\alpha} - \dot{y}_3\right) \\
& +b_4\left(\dot{y}_5 - b\cdot\dot{\theta} + d\cdot\dot{\alpha} - \dot{y}_4\right) - b_6\left(\dot{y}_6 - a\cdot\dot{\theta} - \dot{y}_5 - d\cdot\dot{\alpha}\right) \\
& = u_1 + u_2 + u_3 + u_4
\end{aligned} \tag{3}$$

$$m_6\cdot\ddot{y}_6 + k_6\left(y_6 - y_5 - a\cdot\theta - d\cdot\alpha\right) + b_6\left(\dot{y}_6 - \dot{y}_5 - a\cdot\dot{\theta} - d\cdot\dot{\alpha}\right) = 0 \tag{3}$$

In [18], it was shown that if optimal feedback gains are kept constant then increasing the actuator delay increases the peak value of the sprung mass acceleration accordingly until it runs into the instability, as expected. After the value of $D = 25\,ms$ the quarter-vehicle model is getting unstable. In [29] upper bound of the input delay was also $25\,ms$. [16] Reported that their controller performed well if time delay is smaller than $50\,ms$. In order to show the effects of the actuator delay on the system performance, the time responses of the seat are presented through Figures 2b, 2c, 2d for different widely used control approaches in literature such as State Feedback Control (SFC), Sliding Mode Control (SMC) and Fuzzy Logic Control (FLC), respectively. SFC was chosen as the first controller since it is one of the basic control methods that stability of the controlled system can be investigated easily. SMC was preferred as the second control method for comparison since it is known with its robust character and guaranties system stability. As a third controller for comparison FLC was chosen since it is a model free control method and its design is based on expert knowledge. Details of those controllers are briefly presented in Appendix. All the controlled cases are compared with the passive suspension (uncontrolled) case, and for the active cases the actuator time delay was chosen to be $D = 35\,ms$, which is within the ranges of aforementioned delay values in literature, namely it is between $25 - 50\,ms$.

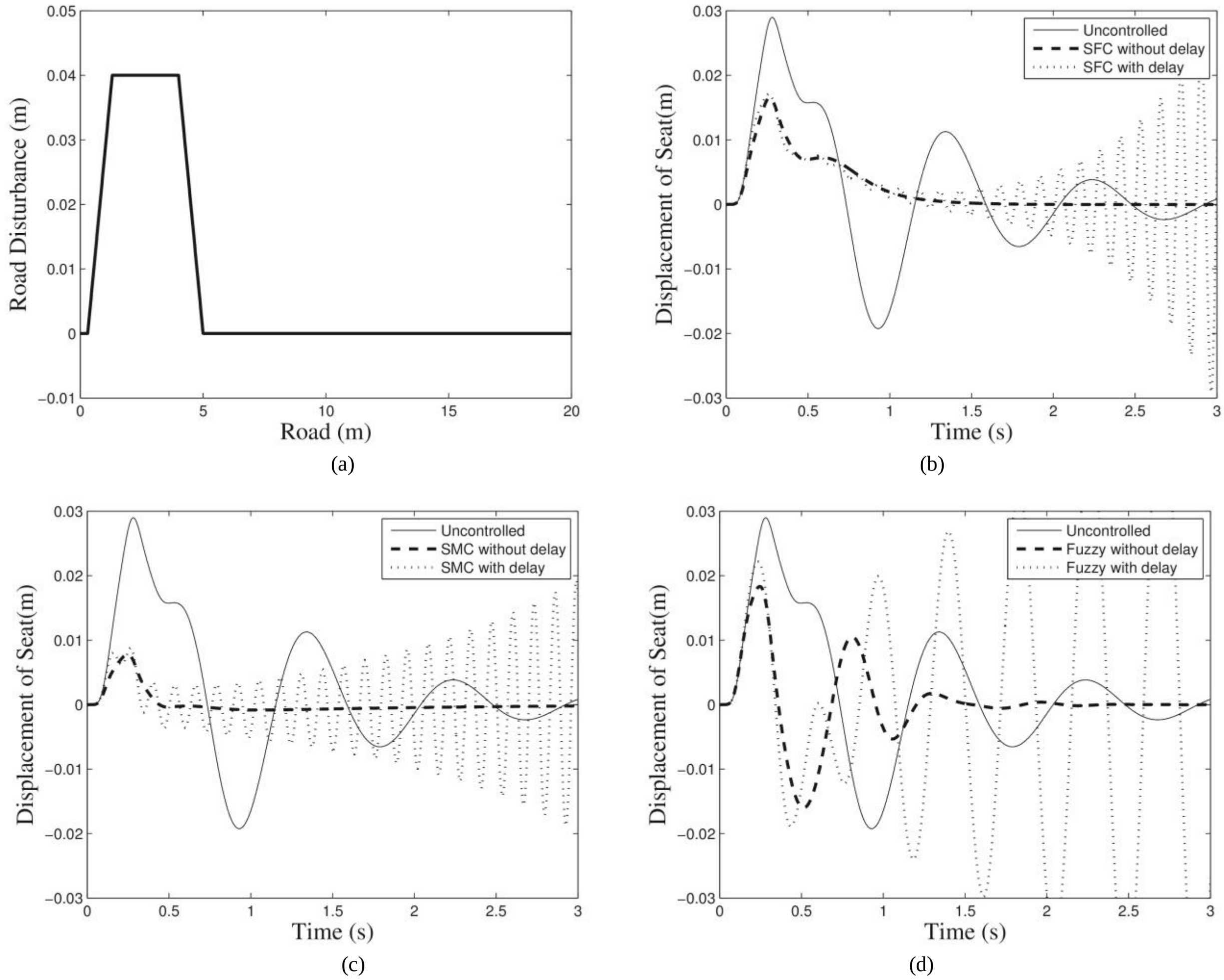


Figure 2. The road input and time responses of the full vehicle model with different controllers: a)The road disturbance acted on the suspension system. b)State feedback controller. c)Sliding mode controller. d)Fuzzy logic controller.

There are two cases for the controlled suspensions namely, the case with time delay and the case without time delay. The case without actuator delay may be thought as the desired force u is produced by the actuator immediately, that is without any time delay. On the other hand in reality it is not possible due to actuator dynamics. For example if a hydraulic servo system is used to generate control force, as presented in work of [17] the delay will be inevitable. Therefore, the performance of these controllers with time delay, $D = 35\,ms$ is also presented here. It is observed from the figures that the SFC, SMC and FLC perform well in the case without actuator delay since displacements of the seat are reduced. On the other hand it is seen that when actuator delay takes place all the controllers that is SFC, SMC and FLC fail to suppress vibrations of the seat and moreover destabilize the vehicle, since the displacements of the seat increased rapidly. Therefore it can be deduced that actuator delay must be taken into account during controller design, which will be the case in this study.

III. Controller Design

The main idea of the backstepping control is to map the investigated system to an easy-to analyze desirable exponentially stable linear system by using new variables on the closed-loop system. Furthermore, the aim of designing backstepping controller for the vehicle active suspension system is to improve ride comfort to an upper level by reducing seat and sprung mass displacement and acceleration magnitudes while considering actuator time delay. As stated before, the actuator time delay is inevitable in practice and as shown in the previous section it will make the active suspension system unstable. Therefore, the effect of actuator time delay will be taken into account in order to assure system stability. Equations of motion of the full vehicle suspension model are presented below in vector matrix form.

$$\frac{d\mathbf{X}}{dt} = \mathbf{AX} + \mathbf{BU}(t-D) + \mathbf{WX_0} \tag{9}$$

where $\mathbf{X}$ is the state vector which is given by

$$\begin{aligned}\mathbf{X} &= \left[x_1 \ x_2 \ .. \ x_{15} \ x_{16} \ \right]^T \\ &= \left[y_1 \ \theta \ \alpha \ y_2 \ y_3 \ y_4 \ y_5 \ y_6 \ \dot{y}_1 \ \dot{\theta} \ \dot{\alpha} \ \dot{y}_2 \ \dot{y}_3 \ \dot{y}_4 \ \dot{y}_5 \ \dot{y}_6 \ \right]^T\end{aligned} \tag{10}$$

The road excitation vector is:

$$\mathbf{X_0} = \left[y_{01}(t) \ y_{02}(t) \ y_{03}(t) \ y_{04}(t)\right]^T$$

where $y_{03}(t) = y_{01}(t-\delta)$, $y_{04}(t) = y_{02}(t-\delta)$ and shown in Figure 2a for the front tires, $\mathbf{U}(t-D)$ is the actuator control signal with time delay, D, and related matrices are presented in Appendix. $(\mathbf{A, B})$ is a controllable pair. By using [21], we modelled the actuator delay by using a first-order hyperbolic partial differential equation

$$u_t(x,t) = u_x(x,t), \tag{11}$$

$$u(D,t) = U(t) \tag{12}$$

which has a solution as $u(x,t) = U(t+x-D)$. Therefore, we get the delayed input as $u(0,t) = U(t-D)$, [21]. By using the following backstepping transformation

$$w(x,t) = u(x,t) - \int_0^x q(x,y)u(y,t)dy - \gamma(x)^T \mathbf{X} \tag{13}$$

which gets system (11)-(12) into the following system

$$\frac{d\mathbf{X}}{dt} = (\mathbf{A} + \mathbf{BK})\mathbf{X} + \mathbf{B}w(0,t) \tag{14}$$

$$w_t(x,t) = w_x(x,t), \tag{15}$$

$$w(0,t) = 0 \tag{16}$$

Here $\mathbf{K}$ is the state feedback control gain vector which stabilizes the system without time delay. To derive necessary functions with straightforward calculations we get the following functions as

$$\gamma(x)^T = \mathbf{K}e^{\mathbf{A}x}, \tag{17}$$

$$q(x,y) = \mathbf{K}e^{\mathbf{A}(x-y)}\mathbf{B} \tag{18}$$

One can see the detailed solution of the function γ and $q(x,y)$ in [21]. Therefore controller for the active suspension system is given by

$$U(D) = \int_0^D \mathbf{K}e^{\mathbf{A}(D-y)}\mathbf{B}u(y,t)dy + \mathbf{K}e^{\mathbf{A}D}\mathbf{X} \tag{19}$$

By using the system (11)-(12) with a transformation the control law for distributed backstepping controller(DBC) can be derived as

$$U(D) = \mathbf{K}\left[e^{\mathbf{A}D}\mathbf{X} + \int_{t-D}^{t} e^{\mathbf{A}(t-\tau)}\mathbf{B}U(\tau)d\tau\right] \tag{20}$$

For the stability analysis following Lyapunov function is defined as [21].

$$\mathbf{V} = \mathbf{X}^T\mathbf{PX} + \frac{a}{2}\int_0^D (1+x)w(x)^2\,dx \tag{21}$$

where $\mathbf{P} = \mathbf{P^T} > 0$ is the solution to the Lyapunov equation $\mathbf{P}(\mathbf{A}+\mathbf{BK}) + (\mathbf{A}+\mathbf{BK})^{\mathbf{T}}\mathbf{P} = \mathbf{-Q}$ for some $\mathbf{Q} = \mathbf{Q^T} > 0$ and $a > 0$. Time derivation of given Lyapunov Function is

$$\begin{aligned}\dot{\mathbf{V}} &= \mathbf{X}^T\left((\mathbf{A}+\mathbf{BK})^{\mathbf{T}}\mathbf{P} + \mathbf{P}(\mathbf{A}+\mathbf{BK})\right)\mathbf{X} \\ &\quad + 2\mathbf{X}^T\mathbf{PB}w(0) - \frac{a}{2}w(0)^2 - \frac{a}{2}\int_0^D w(x)^2\,dx \\ &\le -\mathbf{X}^T\mathbf{QX} + \frac{2}{a}\left\|\mathbf{X}^T\mathbf{PB}\right\|^2 - \frac{a}{2}\int_0^D w(x)^2\,dx\end{aligned} \tag{22}$$

By choosing parameter a properly, stability proof is done. For the detailed stability analysis, [21] can be helpful. During numerical implementation because of the second term of the control law (20), some problems described in [25] such as numerical instabilities can occur. To solve this problem, we use ordinary differential equation

$$\frac{dz}{dt} = \mathbf{A}z + \mathbf{B}u(t) - e^{\mathbf{A}D}\mathbf{B}u(t-D) \tag{23}$$

which has the solution as

$$z(t) = \int_{t-D}^{t} e^{\mathbf{A}(t-\tau)}\mathbf{B}U(\tau)d\tau \tag{24}$$

Since our open-loop system is stable, (24) can be used to calculate the second term in control law (20).

IV. Numerical Results

Numerical results of the designed controller for the full vehicle active suspension system with actuator delay are presented in this section. The actuator time delay applied to the system is $D = 35\,ms$. The time responses for the vehicle body and seat displacements are presented in Figure 3 for the passive suspension, active suspension without delay using SFC and active suspension with delay using designed DBC. When there is no delay the SFC suppresses vehicle vibrations effectively as seen from the figure. It had been shown in Section II that SFC could not cope with actuator time delay that is the suspension system was destabilized. The unstable results for SFC with delay are not presented here to improve the visibility of the figures. When the actuator delay is in effect it is seen from the same figure that the designed DBC stabilizes the system while satisfactorily suppressing the vehicle body displacements.

Since acceleration of the seat, vehicle body heave, pitch and roll motions are also important measure of the ride comfort, they are also presented in Figure 4. If compared with the passive case it is seen that designed controller suppresses the acceleration of the vehicle body and seat which means that the ride comfort is improved.

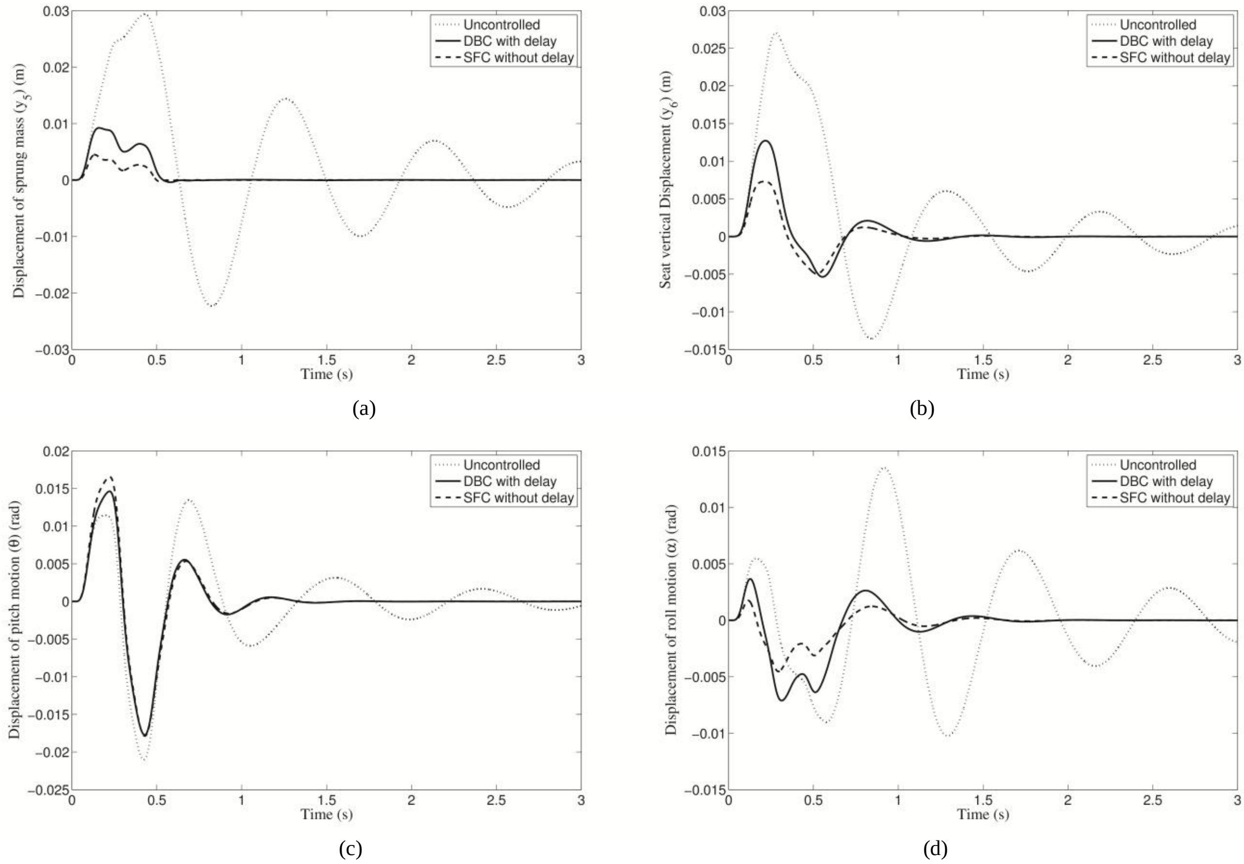
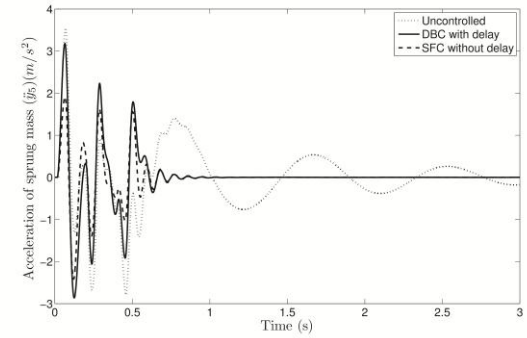
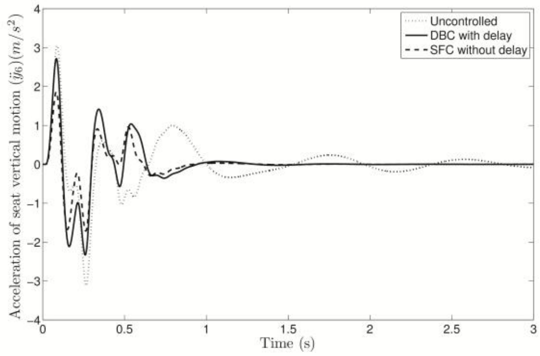


Figure 3. The comparison of the displacements of the full vehicle model and seat for distributed backstepping control and state feedback control: a)Sprung Mass. b)Seat. c)Pitch Motion. d)Roll Motion.



(a)



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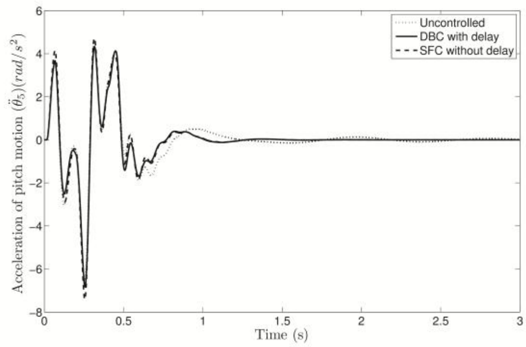

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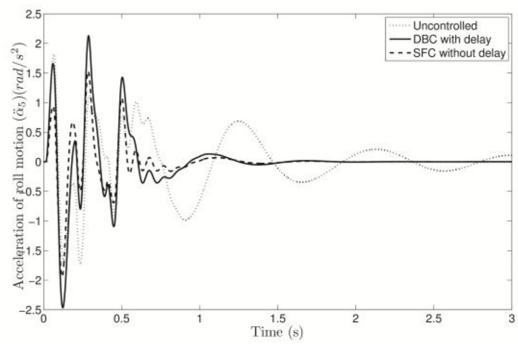
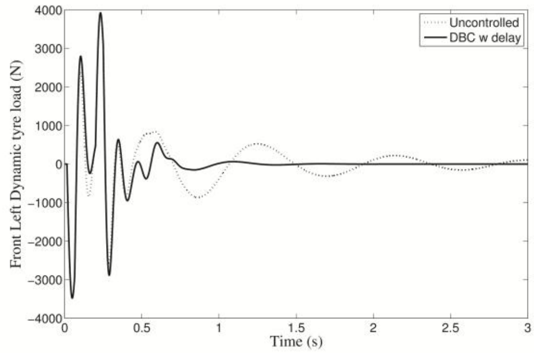
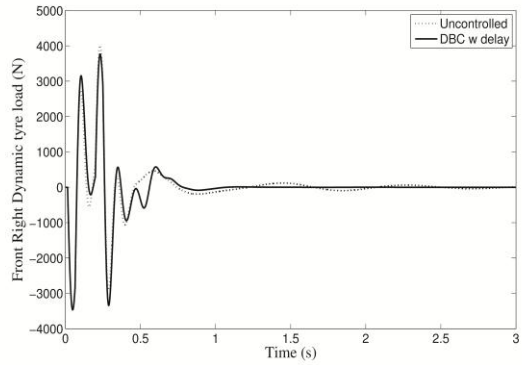


(d)

Figure 4. The comparison of the accelerations of the full vehicle model and seat for distributed backstepping control and state feedback control: a)Sprung Mass. b)Seat. c)Pitch Motion. d)Roll Motion.



(a)



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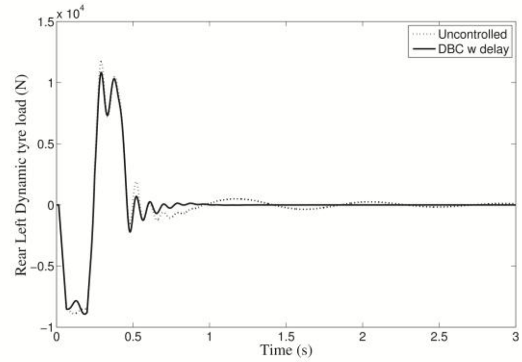

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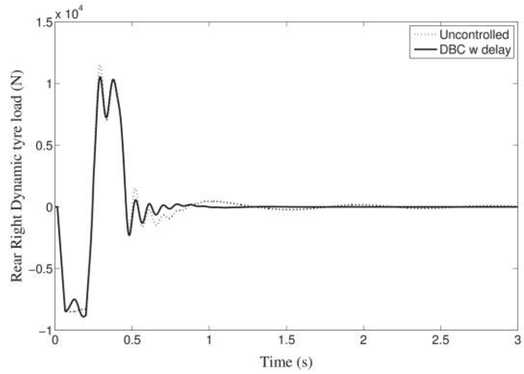
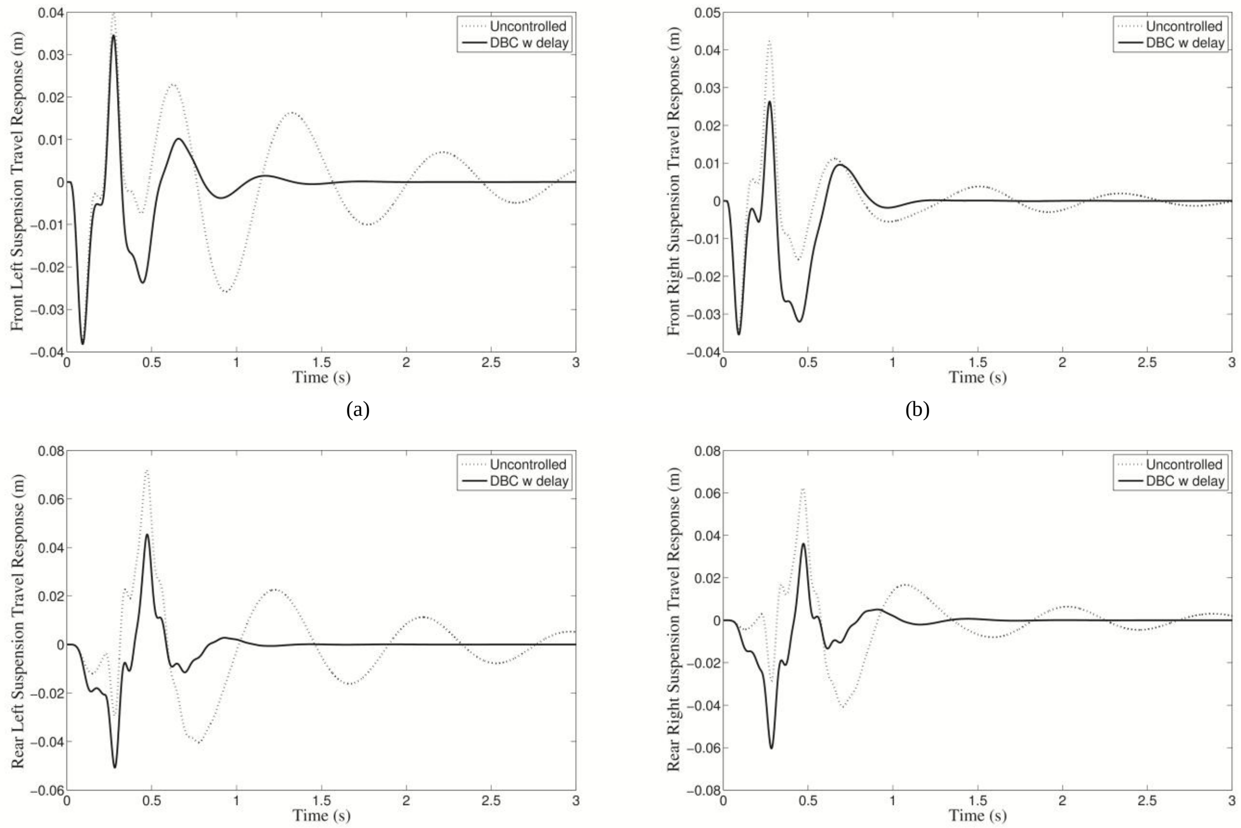


(d)

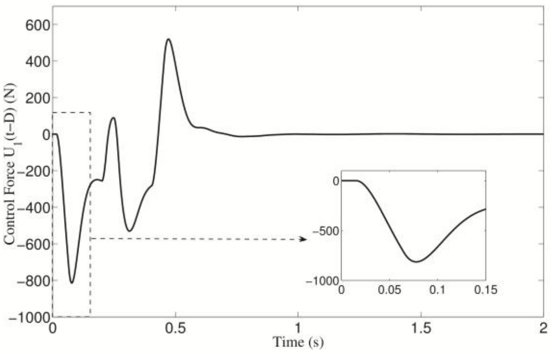
Figure 5. Dynamic tire loads of full vehicle suspension system: a)Left Front. b)Right Front. c)Left Rear. d)Right Rear.

Figure 5 presents the time history of the dynamic tire loads for the full vehicle. It is seen that dynamic tire loads are not increased during ride comfort improvement. Moreover, the dynamic tire loads were also reduced to some degree indicating that road holding was also improved. Suspension travel response of the investigated vehicle active suspension system is presented in Figure 6. It is seen from this figure that the magnitudes of the suspension travel response for the DBC case do not exceed the suspension travel response magnitudes of uncontrolled suspension system. The delayed control signals applied to the full vehicle active suspension is shown in Figure 7. It is possible to observe the time delay effect especially at the beginning on this figure. As a measure of the ride comfort, the root mean square (RMS) values of the acceleration of the seat are presented in Figure 8. It was seen that designed DBC reduced the RMS values if compared with the passive suspension which means that ride comfort was improved. In addition this figure demonstrates the effect of changing actuator time delay. In fact there is not any significant change in RMS values for different time delays, which confirms the success of the designed controller.

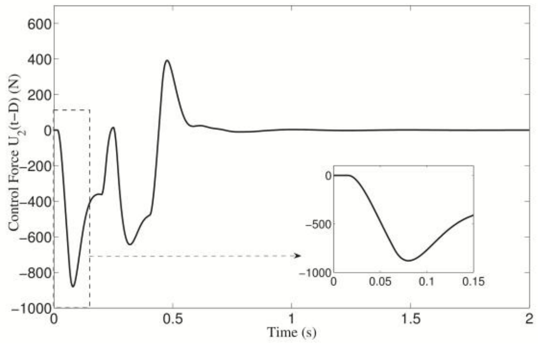


(a) (b) (c) (d)

Figure 6. Suspension travel responses of full vehicle suspension system : a)Left Front. b)Right Front. c)Left Rear. d)Right Rear.



(a)



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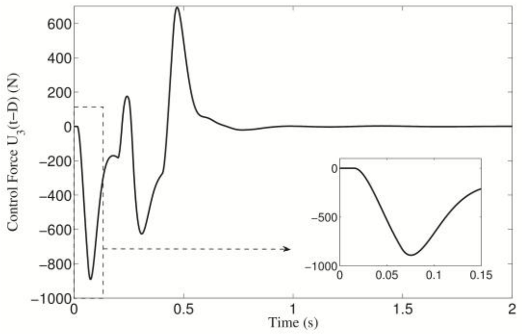
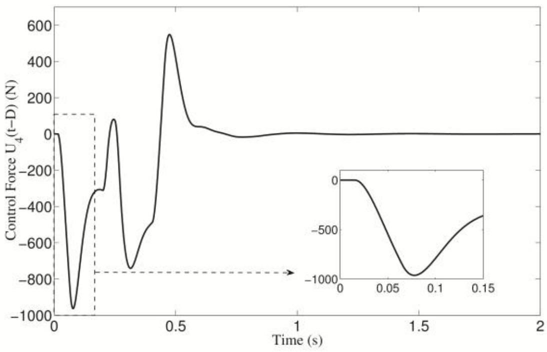


(c)



(d)

Figure 7. Control forces of the vehicle active suspension system: a)Left Front. b)Right Front. c)Left Rear. d)Right Rear.

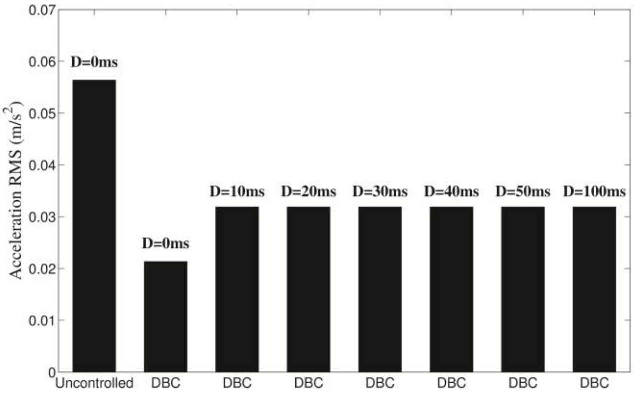


Figure 8. Accelaration RMS of seat by using different delays

V. CONCLUSION

Actuator time delays are inevitable in active suspension systems, and they should be taken into consideration during controller design if not they may give rise to instability of the closed loop system. It has been demonstrated in this paper that many control methods such as state feedback control, sliding mode control and fuzzy logic control could not perform well in the presence of actuator time delay. Therefore in this study, a distributed backstepping controller was designed that has taken into account the actuator time delay by means of first-order hyperbolic partial differential equation. Then this controller was applied to a full vehicle active suspension system with actuator time delay. The time responses have demonstrated that this controller improved ride comfort without reducing road holding of the vehicle along with guaranteed stability of the system.

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APPENDIX

TABLE I. VEHICLE PARAMETERS

m_1	: 66.50 kg	b_2	: 2015 N/m
m_2	: 66.50 kg	b_3	: 935 Ns/m
m_3	: 45.18 kg	b_4	: 935 Ns/m
m_4	: 45.18 kg	b_6	: 500 Ns/m
m_5	: 1380 kg		
m_6	: 28.00 kg	k_1	: 211180 N/m
b_1	: 2015 Ns/m	k_2	: 211180 N/m
k_3	: 211180 N/m	s_z	: 0.785 m
k_4	: 211180 N/m	s_x	: 0.295 m
k_{51}	: 27000 N/m	l_1	: 1.945 m
k_{52}	: 27000 N/m	l_2	: 2.115 m
k_{53}	: 20770 N/m	l_3	: 0.58 m
k_{54}	: 20770 N/m	l_4	: 1.16 m
k_6	: 500.0 N/m	V	: 20 m/s

Control force matrix $[\mathbf{B}]$

$$[\mathbf{B}] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1/m_5 & 1/m_5 & 1/m_5 & 1/m_5 \\ a/I_\theta & a/I_\theta & -\mathrm{b}/I_\theta & -\mathrm{b}/I_\theta \\ -c/I_\alpha & d/I_\alpha & -c/I_\alpha & d/I_\alpha \\ -1/m_1 & 0 & 0 & 0 \\ 0 & -1/m_2 & 0 & 0 \\ 0 & 0 & -1/m_3 & 0 \\ 0 & 0 & 0 & -1/m_4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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