International Journal of
Science and Engineering Investigations vol. 6, issue 68, September 2017
ISSN: 2251-8843

# New Results on Soft Banach Algebra

Seyyed Hossein Jafari Petroudi[1], Seyyed Abbas Sadati[2], Alireza Yaghobi[3]
[1]Assistant Professor, Department of Mathematics, Payame Noor University, P. O. Box 1935-3697, Tehran, Iran
[2,3]Department of Mathematics, Payame Noor University, P. O. Box 1935-3697, Tehran, Iran
([1]petroudi@pnu.ac.ir, [2]sadati_abbas@yahoo.com, [3]ay50math@gmail.com)

***Abstract***- In this paper the ideas of soft spectrum, soft condition spectrum, soft ε_condition spectrum and soft spectral radius of a soft element over soft Banach algebras are introduced. Some basic properties of these ideas in soft Banach algebras are studied. Then we define the notions of soft multiplicative linear functional, almost soft multiplicative linear functional, soft Jordan multiplicative linear functional and almost soft Jordan multiplicative linear functional in soft Banach algebras. Finally some new results and theorems about them over soft Banach algebra are investigated.

***Keywords***- *Condition Spectrum, Soft Spectral Radius, Soft Normed Linear Space, Almost Soft Multiplicative Linear Functional, Soft Jordan Functional*

## I. INTRODUCTION

Due to uncertain data in real world, various problems in mathematics, engineering, environmental sciences, economics and medical sciences cannot be solved by the usual mathematical methods. The difficulty of the usual mathematical method is the lack of the parameterization tools for descriptions of problems arising in the fields of ambiguities and uncertainties.

To dealing with such problems Molodtsov [14] introduced the concept of soft set theory. Soft set theory is an innovative mathematical method which has the capability for dealing with uncertainties. Furthermore it has the parameterization tool which is more flexible than the customary mathematical methods through the vagueness and uncertainties of day to day problems in real world.

Das and Samanta [10] introduced the idea of soft linear functional over soft linear spaces. They studied some basic properties of such operators and extended some fundamental theorems of functional analysis in soft set settings.

Thakur and Samanta [17] introduced the concept of soft Banach algebras and studied some of its preliminary properties. For more information about soft set theory and some of its applications one can see [1-3] and [12-16].

In this paper the ideas of soft spectrum, soft condition spectrum, soft ε_condition spectrum and soft spectral radius of a soft element over soft Banach algebras are introduced. Some basic properties of these ideas in soft Banach algebras are studied. Then we define the notions of soft multiplicative linear functional, almost soft multiplicative linear functional, soft Jordan multiplicative linear functional and almost soft Jordan multiplicative linear functional in soft Banach Algebras. Finally some new results and theorems about them in soft Banach algebras are investigated.

## II. PRELIMINARYE

### A. Soft set, Soft linear operator

#### 1) Definition

Let $U$ be a universe and $E$ be a set of parameters. Let $P(U)$ denote the power set of $U$ and $A$ a non-empty subset of $E$. A pair $(F, A)$ is called a soft set over $U$, where $F$ is a mapping given by $F: A \to P(U)$. In other words, a soft set over $U$ is a parameterized family of subsets of the universe $U$. For $\varepsilon \in A$, $F(\varepsilon)$, maybe considered as the set of $\varepsilon$_ approximate elements of the soft set $(F, A)$. Clearly, a soft set is not a set.

In this study, we shall consider the soft sets with respect to the universal parameter set A.

#### 2) Definition

The complement of a soft set $(\mathrm{F}, \mathrm{A})$ is denoted by $(\mathrm{F}, \mathrm{A})^{\mathrm{C}} = (\mathrm{F}^{\mathrm{C}}, \mathrm{A})$ where, $\mathrm{F}^{\mathrm{C}}: \mathrm{A} \to \mathrm{P}(\mathrm{X})$ is a mapping given by $\mathrm{F}^{\mathrm{C}}(\alpha) = \mathrm{X} - \mathrm{F}(\alpha)$, for all $\alpha \in \mathrm{A}$. Let us call $\mathrm{F}^{\mathrm{C}}$ to be the soft complement function of F.

#### 3) Definition

A soft set $(\mathrm{F}, \mathrm{A})$ over X is said to be a null soft set, denoted by $\Phi$, if for all $\mathrm{e} \in \mathrm{A}, \mathrm{F}(\mathrm{e}) = \emptyset$.

#### 4) Definition

A soft set $(\mathrm{F}, \mathrm{A})$ over X is said to be an absolute soft set, denoted by $\breve{\mathrm{X}}$, if for all all $\mathrm{e} \in \mathrm{A}, \mathrm{F}(\mathrm{e}) = \mathrm{X}$. Clearly, $\breve{\mathrm{X}}^{\mathrm{C}} = \Phi$.

#### 5) Definition

Let X be a nonempty set and $A$ be a nonempty parameter set. Then a function $\varepsilon: A \to \mathrm{X}$ is said to be a soft element of X. A soft element $\varepsilon$ of X is said to belongs to a soft set $B$ of X, which is denoted by $\varepsilon \tilde{\epsilon} B$, if $\varepsilon(e) \in B(e), \forall\, e \,\tilde{\in}\, B$. Thus for a soft set $B$ of X with respect to the index set $B$, we have $B(e) = \{\varepsilon(e), \varepsilon \tilde{\epsilon} B\}, e \epsilon A$.

It is to be noted that every singleton soft set (a soft set $(F, A)$ for which $F(e)$ is a singleton set, $\forall e \in A$ ) can be identified with a soft element by simply identifying the singleton set with the element that it contains,$\forall e \in A$.

*6) Definition*

Let $R$ be the set of real numbers and $\mathcal{B}(R)$ the collection of all non-empty bounded subsets of $R$ and $A$ taken as a set of parameters. Then a mapping $F: A \to B(R)$ is called a soft real set. It is denoted by$(F, A)$. If especially $(F, A)$ is a singleton set, then after identifying $(F, A)$ with the corresponding soft element, it will be called a soft real number.

The set of all soft real numbers is denoted by $\mathbb{R}(A)$ and the set of all non-negative soft real numbers by $\mathbb{R}(A)^*$.

We use notation $\tilde{r}, \tilde{s}, \tilde{t}$ to denote soft real numbers whereas $\bar{r}, \bar{s}, \bar{t}$ will denote a particular type of soft real numbers such that $\bar{r}(\lambda) = r$, for all $\lambda \in A$ etc. For example $\bar{0}$ is the soft real number where $\bar{0}(\lambda)$ , for all $\lambda \in A$.

*7) Definition*

Let $\mathbb{C}$ be the set of complex numbers and $\wp(\mathbb{C})$ be the collection of all nonempty bounded subsets of the set of complex numbers. Also let $A$ be a set of parameters. Then a mapping $F: A \to \wp(\mathbb{C})$ is called a soft complex set. It is denoted by $(F, A)$. If in particular $(F, A)$ is a singleton set, then after identifying $(F, A)$ with the corresponding soft element, it will be called a soft complex number. The set of all soft complex numbers is denoted by $\mathbb{C}(A)$.

*8) Definition*

For two soft real numbers $\tilde{r}, \tilde{s}$ we define

(i) $\tilde{r} \leqq \tilde{s}$ if $\tilde{r}(\lambda) \leqq \tilde{s}(\lambda)$, for all $\lambda \in A$.

(ii) $\tilde{r} \geqq \tilde{s}$ if $\tilde{r}(\lambda) \geqq \tilde{s}(\lambda)$, for all $\lambda \in A$.

(iii) $\tilde{r} \lessapprox \tilde{s}$ if $\tilde{r}(\lambda) \lessapprox \tilde{s}(\lambda)$, for all $\lambda \in A$.

(iv) $\tilde{r} \gtrapprox \tilde{s}$ if $\tilde{r}(\lambda) \gtrapprox \tilde{s}(\lambda)$, for all $\lambda \in A$.

*9) Definition*

Let $(F, A)$ , $(G, A) \in \mathbb{C}(A)$ . Then the sum, difference, product and division are defined by

$(F + G)(\lambda) = z + w; z \in F(\lambda), w \in G(\lambda), \forall \lambda \in A;$

$(F - G)(\lambda) = z - w;\ z \in F(\lambda), w \in G(\lambda), \forall \lambda \in A;$

$(FG)(\lambda) = zw; z \in F(\lambda), w \in G(\lambda), \forall \lambda \in A;$

$(F/G)(\lambda) = z/w; z \in F(\lambda), w \in G(\lambda), \forall \lambda \in A;$
provided $G(\lambda) \neq \emptyset$.

*10) Definition*

Let $(F, A)$ be a soft complex number. Then the modulus of $(F, A)$ is denoted by $(|F|, A)$ and is defined by $|F|(\lambda) = |z|; z \in F(\lambda), \forall \lambda \in A$ where $z$ is an ordinary complex number.

Since the modulus of each ordinary complex number is a non-negative real number and by definition of soft real numbers it follows that $(|F|, A)$ is a non-negative soft real number for every soft complex number$(F, A)$.

Let $X$ be a non-empty set. Let $\check{X}$ be the absolute soft set i.e., $(\lambda) = X, \ \forall \lambda \in A$ , where $(F, A) = \check{X}$ . Let $\mathcal{S}(\check{X})$ be the collection of all soft sets $(F, A)$ over $X$ for which $F(\lambda) \neq \emptyset$, for all $\lambda \in A$ together with the null soft set $\Phi$.

Let $(F, A)(\neq \emptyset) \in \mathcal{S}(\check{X})$ , then the collection of all soft elements of $(F, A)$ will be denoted by $SE(F, A)$ . For a collection $\boldsymbol{\mathcal{B}}$ of soft elements of $\check{X}$, the soft set generated by $\boldsymbol{\mathcal{B}}$ is denoted by $SS(\boldsymbol{\mathcal{B}})$.

*11) Definition*

A mapping $d: SE(\check{X}) \times SE(\check{X}) \to \mathbb{R}(A)^*$ , is said to be a soft metric on the soft set $\check{X}$ if $d$ satisfies the following conditions:

$(M1)$ $d(\tilde{x}, \tilde{y}) \geqq \bar{0}$, for all $\tilde{x}, \tilde{y} \mathrel{\tilde{\in}} \check{X}$.

$(M2)$ $d(\tilde{x}, \tilde{y}) = \bar{0}$ if and only if $\tilde{x} = \tilde{y}$.

$(M3)$ $d(\tilde{x}, \tilde{y}) = d(\tilde{y}, \tilde{x})$, for all$\tilde{x}, \tilde{y} \mathrel{\tilde{\in}} \check{X}$.

(M4) For all $\tilde{x}, \tilde{y} \mathrel{\tilde{\in}} \check{X}, d(\tilde{x}, \tilde{z}) \leq d(\tilde{x}, \tilde{y}) + d(\tilde{y}, \tilde{z})$.

The soft set $\check{X}$ with a soft metric $d$ on $\check{X}$ is said to be a soft metric space and is denoted by $(\check{X}, d, A)$ or $(\check{X}, d)$.

*12) Theorem (Decomposition theorem)*

If a soft metric $d$ satisfies the condition:

(M5) For $(\xi, \eta) \epsilon X \times X$, and $\lambda \in A$, $\{d(\tilde{x}, \tilde{y})(\lambda): (\tilde{x})(\lambda) = \xi,\ (\tilde{y})(\lambda) = \eta\}$ is a singleton set, and if for $\lambda \in A, d_\lambda: X \times X \to \mathbb{R}^+$ is defined by $d_\lambda(\xi, \eta) = d(\tilde{x}, \tilde{y})(\lambda)$ , where $\tilde{x}, \tilde{y} \in \check{X}$ such that $(\tilde{x})(\lambda) = \xi,\ (\tilde{y})(\lambda) = \eta$.Then $d_\lambda$ is a metric on $X$.

*13) Definition*

Let $(\check{X}, d)$ be a soft metric space, $\tilde{r}$ be a non-negative soft real number and $\tilde{a} \in \check{X}$. By an open ball with center $\tilde{a}$ and radius $\tilde{r}$ , we mean the collection of soft elements of $\check{X}$ satisfying $d(\tilde{x}, \tilde{a}) \leqq \tilde{r}$. The open ball with center $\tilde{a}$ and radius $\tilde{r}$ is denoted by $B(\tilde{a}, \tilde{r})$. Thus $B(\tilde{a}, \tilde{r}) = \{\tilde{x} \mathrel{\tilde{\in}} \check{X}: d(\tilde{x}, \tilde{a}) \lessapprox \tilde{r}\} \subset SE(\check{X})$.

$SS(B(\tilde{a}, \tilde{r}))$ will be called a soft open ball with center $\tilde{a}$ and radius $\tilde{r}$.

*14) Definition*

Let $\boldsymbol{\mathcal{B}}$ be a collection of soft elements of $\check{X}$ in a soft metric space $(\check{X}, d)$.Then a soft element $\tilde{a}$ is said to be an interior element of $\boldsymbol{\mathcal{B}}$ if there exists a positive soft real number $\tilde{r}$ such that $\tilde{a} \in B(\tilde{a}, \tilde{r}) \subset \boldsymbol{\mathcal{B}}$.

*15) Definition*

Let $(Y, A)$ be a soft subset in a soft metric space $(\check{X}, d)$. Then a soft element $\tilde{a}$ is said to be an interior element of $(Y, A)$ if there exists a positive soft real number $\tilde{r}$ such that $\tilde{a} \in B(\tilde{a}, \tilde{r}) \subset SE(Y, A)$.

*16) Definition*

Let $(\check{X}, d)$ be a soft metric space and $\boldsymbol{\mathcal{B}}$ be a non-null collection of soft elements of $\check{X}$.Then $\boldsymbol{\mathcal{B}}$ is said to be‘open in $\check{X}$ with respect to $d$ ’or‘open in $(\check{X}, d)$’ if all elements of $\boldsymbol{\mathcal{B}}$ are interior elements of $\boldsymbol{\mathcal{B}}$.

*17) Definition*

Let $(\breve{X}, d)$ be a soft metric space and $(Y, A)$ be a non-null soft subset $\in S(\breve{X})$ in $\breve{X}$. Then $(Y, A)$ is said to be 'soft open in $\breve{X}$ with respect to $d$' if there is a collection $\boldsymbol{\mathcal{B}}$ of soft elements of $(Y, A)$ such that $\boldsymbol{\mathcal{B}}$ is open with respect to $d$ and $(Y, A) = SS(\boldsymbol{\mathcal{B}})$.

*18) Definition*

Let $(\breve{X}, d)$ be a soft metric space. A soft set $(Y, A) \in S(\breve{X})$ is said to be 'soft closed in $\breve{X}$ with respect to $d$' if its complement $(Y, A)^c$ is a member of $S(\breve{X})$ and is soft open in $(\breve{X}, d)$.

*19) Definition*

Let $(\breve{X}, d)$ be a soft metric space satisfies (M5). Then $(F, A)$ is soft open with respect to $d$ if and only if $(F, A)(\lambda)$ is open in $(X, d_\lambda)$, for each $\lambda \in A$.

*20) Definition*

Let $V$ be a vector space over a field $K$ and let $A$ be a parameter set. Let $G$ be a soft set over $(V, A)$. Now $G$ is said to be a soft vector space or soft linear space of $V$ over $K$ if $G(\lambda)$ is a vector subspace of $V, \forall \lambda \in A$.

*21) Definition*

Let $G$ be a vector space of $V$ over $K$. Then a soft element of $G$ is said to be a soft vector of $G$. In a similar manner a soft element of the soft set $(K, A)$ is said to be a soft scalar, $K$ being the scalar field.

*22) Definition*

Let $\tilde{x}, \tilde{y}$ be soft vectors of $G$ and $\tilde{k}$ be a soft scalar. Then the addition $\tilde{x} + \tilde{y}$ and scalar multiplication $\tilde{k}.\tilde{x}$ of $\tilde{k}$ and $\tilde{x}$ are defined by $(\tilde{x} + \tilde{y})(\lambda) = \tilde{x}(\lambda) + \tilde{y}(\lambda)$, $(\tilde{k}.\tilde{x})(\lambda) = \tilde{k}(\lambda).\tilde{x}(\lambda), \forall \lambda \in A$. Obviously $\tilde{x} + \tilde{y}, \tilde{k}.\tilde{x}$ are soft vectors of $G$.

*23) Definition*

Let $\breve{X}$ be the absolute soft vector space i.e, $\breve{X}(\lambda) = X$, $\forall \lambda \in A$. Then a mapping $\|\cdot\|: SE(\breve{X}) \to \mathbb{R}(A)^*$ is said to be soft norm on the soft vector space $\breve{X}$ if $\|\cdot\|$ satisfies the following conditions:

$(N1)$. $\|\tilde{x}\| \geqq \bar{0}$, for all $\tilde{x} \tilde{\in} \breve{X}$.

$(N2)$. $\|\tilde{x}\| = \bar{0}$ if and only if $\tilde{x} = \Theta$.

$(N3)$. $\|\tilde{\alpha}\tilde{x}\| \geqq |\tilde{\alpha}|\|\tilde{x}\|$, For all $\tilde{x} \tilde{\in} \breve{X}$ and for every soft scalar $\tilde{\alpha}$.

$(N4)$. For all $\tilde{x}, \tilde{y} \tilde{\in} \breve{X}$, $\|\tilde{x} + \tilde{y}\| \leqq \|\tilde{x}\| + \|\tilde{y}\|$.

The soft vector space $\breve{X}$ with a soft norm $\|\cdot\|$ on $\breve{X}$ is said to be a soft normed linear space and is denoted by $(\breve{X}, \|\cdot\|, A)$ or $(\breve{X}, \|\cdot\|)$. $(N1), (N2)$, $(N3)$ and $(N4)$ are said to be soft norm axioms.

*24) Example*

Let $\mathbb{R}(A)$ be the set of all soft real numbers. Then the mapping $\|\cdot\|: \mathbb{R}(A) \to \mathbb{R}(A)^*$ which is defined by $\|\tilde{x}\| = |\tilde{x}|$, for all $\tilde{x} \in \mathbb{R}(A)$, where $|\tilde{x}|$ denotes the modulus of soft real numbers, is a soft norm on $\mathbb{R}(A)$ and since $SS(\mathbb{R}(A)) = \breve{\mathbb{R}}$, thus $(\breve{\mathbb{R}}, \|\cdot\|, A)$ or $(\breve{\mathbb{R}}, \|\cdot\|)$ is a soft normed linear space. With the same argument $SS(\mathbb{C}(A)) = \breve{\mathbb{C}}$ is also a soft normed linear space.

*25) Proposition*

Let $(\breve{X}, \|\cdot\|, A)$ be a soft normed linear space. Then the mapping $d: \breve{X} \times \breve{X} \to \mathbb{R}(A)^*$ which is defined by $d(\tilde{x}, \tilde{y}) = \|\tilde{x} - \tilde{y}\|$, for all $\tilde{x}, \tilde{y} \in \breve{X}$, is a soft metric on $\breve{X}$.

*26) Theorem*

Suppose a soft norm satisfies the condition:

$(N5)$. For $\xi \in X$, and $\lambda \in A$, $\{\|\tilde{x}\|(\lambda): \tilde{x}(\lambda) = \xi\}$ is a singleton set.

Then for each $\lambda \in A$, the mapping $\|\cdot\|_\lambda: X \to R^+$ defined by $\|\xi\|_\lambda = \|\tilde{x}\|(\lambda)$, for all $\xi \in X$ and $\tilde{x} \tilde{\in} \breve{X}$ such that $\tilde{x}(\lambda) = \xi$, is a norm on X.

*27) Proposition*

Let $(\breve{X}, \|\cdot\|, A)$ be a soft normed linear space satisfying (N5), then the induced soft metric $d: \breve{X} \times \breve{X} \to \mathbb{R}(A)^*$ by $d(\tilde{x}, \tilde{y}) = \|\tilde{x} - \tilde{y}\|$, for all $\tilde{x}, \tilde{y} \in \breve{X}$; satisfies (M5).

*28) Definition*

A sequence of soft elements $\{\tilde{x}_n\}$ in a soft normed linear space $(\breve{X}, \|\cdot\|, A)$ is said to be convergent and converges to a soft element $\tilde{x}$ if $\|\tilde{x}_n - \tilde{x}\| \to \bar{0}$ as $n \to \infty$. This means for every $\tilde{\epsilon} \geqq \bar{0}$, chosen arbitrary, there exists a natural number $= N(\epsilon)$, such that $\bar{0} \leqq \|\tilde{x}_n - \tilde{x}\| \lesssim \tilde{\epsilon}$, whenever $n > N$. We denote this by $\tilde{x}_n \to \tilde{x}$ as $n \to \infty$ or by $\lim_{n\to\infty} \tilde{x}_n = \tilde{x}$. $\tilde{x}$ is said to be the limit of the sequence $\tilde{x}_n$ as $n \to \infty$.

*29) Definition*

A sequence $\{\tilde{x}_n\}$ in a soft normed linear space $(\breve{X}, \|\cdot\|, A)$ is said to be Cauchy sequence in $\breve{X}$ if corresponding to every $\tilde{\epsilon} \geqq \bar{0}$, there exists $m \in N$ such that

$$\|\tilde{x}_i - \tilde{x}_j\| \lesssim \tilde{\epsilon}, \forall i, j \geq m, \text{ i.e., } \|\tilde{x}_i - \tilde{x}_j\| \to \bar{0} \text{ as } i, j \to \infty.$$

*30) Definition*

Let $(\breve{X}, \|\cdot\|, A)$ be a soft normed linear space. Then $\breve{X}$ is said to be complete if every Cauchy sequence in $\breve{X}$ convergents to a soft element of $\breve{X}$. Every complete soft normed linear space is called a soft Banach space.

*31) Theorem*

Every Cauchy sequence in $\mathbb{R}(A)$, where $A$ is a finite set of parameters, is convergent, i.e., the set of all soft real numbers with its usual modulus soft norm with Respect to a finite set of parameters, is a soft Banach space.

*32) Definition*

Let $\{\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n\}$ be a set of soft vectors of a soft vector space $G$ such that $\tilde{\alpha}_i(\lambda) \neq \theta$ for any $\lambda \in A$ and $i = 1,2, \ldots, n$. Then $\{\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n\}$ is said to be linearly Independent in $G$ if for any set of soft scalar $\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_n$, $\tilde{c}_1 + \tilde{c}_2 + \cdots + \tilde{c}_n = \Theta$, implies $\tilde{c}_1 = \tilde{c}_2 = \cdots = \tilde{c}_n = \bar{o}$.

*33) Proposition*

A set $S = \{\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n\}$ of soft vectors in a soft vector space $G$ over $V$ is linearly independent if and only if the sets $S(\lambda) = \{\tilde{\alpha}_1(\lambda), \tilde{\alpha}_2(\lambda), \ldots, \tilde{\alpha}_n(\lambda)\}$ are linearly Independent in $V, \forall \lambda \in A$.

*34) Definition*

A soft linear space $\check{X}$ is said to be of finite dimensional if there is a finite set of linearly independent soft vectors in $\check{X}$ which also generates $\check{X}$, i.e any soft element of $\check{X}$ can be expressed as a linear combination of those linearly independent soft vectors. The set of those linearly independent soft vectors is said to be the basis for $\check{X}$ and the number of soft vectors of the basis is called the dimension of $\check{X}$.

*35) Definition*

A soft subset $(Y, A)$ with $Y(\lambda) \neq \emptyset$, $\forall \lambda \in A$, in a soft normed linear space $(\check{X}, \|\cdot\|, A)$ is said to be bounded if there exists a soft real number $\tilde{k}$ such that $\|\tilde{x}\| \mathrel{\tilde{\leqq}} \tilde{k}$, $\forall \tilde{x} \in (Y, A)$.

*36) Definition*

A sequence of soft real number $\{\tilde{s}_n\}$ is said to be convergent if for arbitrary $\tilde{\varepsilon} \mathrel{\tilde{>}} \bar{o}$, there exists a natural number $N$ such that for all $n \geq N$, $|\tilde{s} - \tilde{s}_n| \mathrel{\tilde{<}} \tilde{\varepsilon}$. We denote it by $\lim_{n\to\infty} \tilde{s}_n = \tilde{s}$.

*37) Definition*

Let $T: SE(\check{X}) \to SE(\check{Y})$ be an operator. Then $T$ is said to be soft linear if

$(L1)$. $T$ is additive, i.e, $T(\tilde{x}_1 + \tilde{x}_2) = T(\tilde{x}_1) + T(\tilde{x}_2)$ for every soft elements $\tilde{x}_1, \tilde{x}_2 \mathrel{\tilde{\in}} \check{X}$.

$(L2)$. $T$ is homogeneous, i.e., for every soft scalar $\tilde{c}$, $T(\tilde{c}\tilde{x}) = \tilde{c}T(\tilde{x})$, for every soft elements $\tilde{x} \mathrel{\tilde{\in}} \check{X}$.

For every soft elements $\tilde{x}_1, \tilde{x}_2 \mathrel{\tilde{\in}} \check{X}$, the properties $(L1)$ and $(L2)$ can be put in a combined form

$T(\tilde{c}_1\tilde{x}_1 + \tilde{c}_2\tilde{x}_2) = \tilde{c}_1 T(\tilde{x}_1) + \tilde{c}_2 T(\tilde{x}_2)$.

*38) Definition*

The operator $T: SE(\check{X}) \to SE(\check{Y})$ is said to be continuous at $\tilde{x}_o \mathrel{\tilde{\in}} \check{X}$ if for every sequence $\{\tilde{x}_n\}$ of soft elements of $\check{X}$ with $\tilde{x}_n \to \tilde{x}_o$ as $n \to \infty$, we have $T(\tilde{x}_n) \to T(\tilde{x}_o)$ as $n \to \infty$ i.e. , $\|\tilde{x}_n \to \tilde{x}_o\| \to \bar{o}$ as $n \to \infty$ implies $\|T(\tilde{x}_n) - T(\tilde{x}_o)\| \to \bar{o}$ as $n \to \infty$. If $T$ is continuous at each soft element of $\check{X}$, then $T$ is said to be continuous operator.

*39) Definition*

Let $T: SE(\check{X}) \to SE(\check{Y})$ be a soft linear operator, where $\check{X}$, $\check{Y}$ are soft normed linear space. The operator $T$ is called bounded if there exists some positive soft real number $\widetilde{M}$ such that for all $\tilde{x} \mathrel{\tilde{\in}} \check{X}$, $\|T(\tilde{x}))\| \mathrel{\tilde{\leqq}} \widetilde{M}\|\tilde{x}\|$.

*40) Theorem*

Let $T: SE(\check{X}) \to SE(\check{Y})$ be a soft linear operator, where $\check{X}$, $\check{Y}$ are soft normed linear space. If $T$ is a bounded then $T$ is continuous.

*41) Theorem*

Decomposition theorem) Suppose a soft linear operator $T: SE(\check{X}) \to SE(\check{Y})$ ,where $\check{X}$, $\check{Y}$ are soft normed linear Space, satisfies the condition $(L3)$. For $\in X$, and $\lambda \in A$, $\{T(\tilde{x})(\lambda): \tilde{x} \mathrel{\tilde{\in}} \check{X}\ such\ that\ \tilde{x}(\lambda) = \xi\}$ is a singleton set. Then for each $\lambda \in A$, the mapping $T_\lambda: X \to Y$ defined by $T_\lambda(\xi) = T(\tilde{x})(\lambda)$, for all $\xi \in X$ and $\tilde{x} \mathrel{\tilde{\in}} \check{X}$ such that $\tilde{x}(\lambda) = \xi$, is a linear operator.

*42) Theorem*

Let $\check{X}, \check{Y}$ be soft normed linear Spaces which satisfy (N5) and $T: SE(\check{X}) \to SE(\check{Y})$ be a soft linear operator satisfying$(L3)$. If $T$ is continuous then $T$ is bounded.

*43) Definition*

Let $T$ be a bounded soft linear operator from $SE(\check{X})$ into $SE(\check{Y})$. Then the norm of the operator $T$ denoted by $\|T\|$, is a soft real number defined as the following:

For each $\lambda \in A$, $\|T\|(\lambda) =$

$inf\{t \in \mathbb{R}; \|T(\tilde{x})\|(\lambda) \leq t. \|\tilde{x}\|(\lambda), for\ each\ \tilde{x} \mathrel{\tilde{\in}} \check{X}\}$.

*44) Theorem*

Let $\check{X}$, $\check{Y}$ be soft normed linear Spaces which satisfy (N5) and $T$ satisfy $(L3)$. Then for each $\lambda \in A$, $\|T\|(\lambda) = \|T_\lambda\|_\lambda$, where $\|T_\lambda\|_\lambda$ is the norm of the linear operator $T_\lambda: X \to y$.

*45) Theorem*

$\|T(\tilde{x})\| \mathrel{\tilde{\leqq}} \|T\|\|\tilde{x}\|$, for all $\tilde{x} \mathrel{\tilde{\in}} \check{X}$.

*46) Theorem*

Let $\check{X}$, $\check{Y}$ be soft normed linear Spaces which satisfy (N5) and $T: SE(\check{X}) \to SE(\check{Y})$ be a soft linear operator satisfying $(L3)$. Then

(i). $\|T\|(\lambda) = sup\{\|T(\tilde{x})\|(\lambda): \|\tilde{x}\| \mathrel{\tilde{\leqq}} \bar{1}\} = \|T_\lambda\|_\lambda$, for each $\lambda \in A$.

(ii). $\|T\|(\lambda) = sup\{\|T(\tilde{x})\|(\lambda): \|\tilde{x}\| = \bar{1}\} = \|T_\lambda\|_\lambda$, for each $\lambda \in A$.

(iii). $\|T\|(\lambda) = sup\left\{\frac{\|T(\tilde{x})\|}{\|\tilde{x}\|}(\lambda): \|\tilde{x}\|(\mu) \neq o, for\ all\ \mu \in A\right\} = \|T_\lambda\|_\lambda$, for each $\lambda \in A$.

*47) Theorem*

Let $\check{X}$, $\check{Y}$ be soft normed linear Spaces which satisfy (N5). Let $T: SE(\check{X}) \to SE(\check{Y})$ be a continuous soft linear operator satisfying$(L3)$. Then $T_\lambda$ is continuous on $X$ for each $\lambda \in A$.

*48) Theorem*

Let $\check{X}, \check{Y}$ be soft normed linear Spaces which satisfy (N5). Let $\{T_\lambda; \lambda \in A\}$ be a family of bounded linear operators such that $T_\lambda: X \to Y$ for each $\lambda$. Then the soft linear operator $T: SE(\check{X}) \to SE(\check{Y})$ defined by $(T(\tilde{x}))(\lambda) = T_\lambda(\tilde{x}(\lambda))$ $\forall \lambda \in A$, is a bounded soft linear operator satisfying $(L3)$.

*49) Definition*

An operator $T: SE(\breve{X}) \to SE(\breve{Y})$ is called injective or one-to-one if $T(\tilde{x}_1) = T(\tilde{x}_2)$ implies $\tilde{x}_1 = \tilde{x}_2$ . It is called surjective or onto if $R(T) = SE(\breve{Y})$. The operator $T$ is bijective if $T$ is both injective and surjective.

*B. Soft linear functional*

*1) Definition*

A soft linear functional f is a soft linear operator such that $f: SE(\breve{X}) \to K$ where $\breve{X}$ is a soft linear space and $K = \mathbb{R}(A)$ if $\breve{X}$ is a real soft linear space and $K = \mathbb{C}(A)$ if $\breve{X}$ is a complex soft linear space.

Since $SS(\mathbb{R}(A)) = \breve{\mathbb{R}}$ and $SS(\mathbb{C}(A)) = \breve{\mathbb{C}}$ are soft normed linear space, the definition and theorem for soft linear operator over soft normed linear space remain true for soft linear functional.

*2) Theorem*

If a soft linear functional f is continuous at some soft element $\tilde{x}_0 \in \breve{X}$ then f is continuous at every soft element of $\breve{X}$.

*3) Definition*

The soft linear functional f is called bounded if there exists some positive Soft real number $\widetilde{M}$ such that for all $\tilde{x} \,\tilde{\in}\, \breve{X}$, $\|f(\tilde{x})\| \,\tilde{\leq}\, \widetilde{M}\|\tilde{x}\|$ .

*4) Theorem*

A soft linear functional f is continuous if it is bounded.

*5) Theorem*

Let $\breve{X}$ be a soft normed linear space which satisfy (N5) and $f: SE(\breve{X}) \to K$ be a soft linear functional satisfy(L3). If f is continuous then f is bounded.

*6) Theorem*

Let $\breve{X}$ be a soft normed linear space which satisfy (N5) and $f: SE(\breve{X}) \to K$ be a soft linear functional. If $\breve{X}$ is of finite dimension, then f is bounded and hence is continuous.

*7) Definition*

Let f be a bounded soft linear functional. Then the norm of the functional f denoted by $\|f\|$, is a soft real number defined as the following.

For each $\lambda \in A$,

$$\|f\|(\lambda) = \inf\{t \in R; \|f(\tilde{x})\|(\lambda) \leq t. \|(\tilde{x})\|(\lambda), \text{ for all } \tilde{x} \,\tilde{\in}\, \breve{X}\}.$$

*8) Theorem*

Let $\breve{X}$ be a soft normed linear space which satisfy (N5) and f satisfy (L3), then for each $\lambda \in A$, $\|f\|(\lambda) = \|f_\lambda\|_\lambda$ , where $\|f_\lambda\|_\lambda$ is the norm of the norm of the linear functional $f_\lambda$ on X.

*9) Theorem*

$\|f(\tilde{x})\| \,\tilde{\leq}\, \|f\|\|\tilde{x}\|$, for all $\tilde{x} \,\tilde{\in}\, \breve{X}$.

*10) Theorem*

Let $\breve{X}$ be a soft normed linear space which satisfy (N5) and $f: SE(\breve{X}) \to K$ be a soft linear functional on $\breve{X}$ satisfying (L3). Then

(i). $\|f\|(\lambda) = \sup\{ \|f(\tilde{x})\|(\lambda): \|\tilde{x}\| \,\tilde{\leq}\, \bar{1} \} = \|f_\lambda\|_\lambda$ , for each $\lambda \in A$,

(ii). $\|f\|(\lambda) = \sup\{ \|f(\tilde{x})\|(\lambda): \|\tilde{x}\| = \bar{1} \} = \|f_\lambda\|_\lambda$ , for each $\lambda \in A$,

(iii). $\|f\|(\lambda) = \sup\left\{\frac{\|f(\tilde{x})\|}{\|\tilde{x}\|}(\lambda) : \|\tilde{x}\|(\mu) \neq \bar{1}, \text{for all } \mu \in A\right\} = \|f_\lambda\|_\lambda$ , for each $\lambda \in A$.

*11) Theorem*

Let $\breve{X}$ be a soft normed linear space which satisfy(N5). Let $f: SE(\breve{X}) \to K$ be a continuous soft linear functional on $\breve{X}$ satisfying(L3). Then $f_\lambda$ is continuous on $\breve{X}$ for each $\lambda \in A$.

*12) Theorem*

Let $\breve{X}$ be a soft normed linear space which satisfy(N5). Let $\{f_\lambda, \lambda \in A\}$ be a family of continuous linear functionals such that $f_\lambda: X \to \mathbb{R}$ or $\mathbb{C}$ for each $\lambda$. Then the functional $f: SE(\breve{X}) \to K(= \mathbb{R}(A) \text{ or } \mathbb{C}(A))$ define by $(f(\tilde{x}))(\lambda) = f_\lambda(\tilde{x}(\lambda))$, $\forall \lambda \in A$ and $\forall \tilde{x} \,\tilde{\in}\, \breve{X}$, is a continuous soft linear functional satisfying (L3).

*C. Soft Banach algebra*

*1) Definition*

Let V be an algebra over a field $\mathbb{C}$ of complex numbers and let A be the parameter set and $(G, A)$ be a soft set over V. Now $(G, A)$ is said to be a soft algebra of V over $\mathbb{C}$ if $G(\lambda)$ is a subalgebra of V, $\forall \lambda \in A$.

If $(G, A)$ is a soft Banach space with respect to a soft norm that satisfies the inequality $\|\tilde{x}\tilde{y}\| \,\tilde{\leq}\, \|\tilde{x}\|\|\tilde{y}\|$ and if $(G, A)$ contains an identity $\tilde{e}$ such that $\tilde{x}\tilde{e} = \tilde{e}\tilde{x} = \tilde{x}$ with $\|\tilde{e}\| = \bar{1}$, then $(G, A)$ is called a soft Banach algebra. In addition, if in a soft Banach algebra$(G, A)$, $\tilde{x}\tilde{y} = \tilde{y}\tilde{x}$, $\forall \tilde{x}, \tilde{y} \,\tilde{\in}\, \breve{G}$ then $(G, A)$ is called a commutative soft Banach algebra.

*2) Proposition*

$(G, A)$ is a soft Banach algebra iff $G(\lambda)$ is a Banach algebra $\forall \lambda \in A$.

*3) Proposition*

In a soft Banach algebra if $\tilde{x}_n \to \tilde{x}$ and $\tilde{y}_n \to \tilde{y}$ then $\tilde{x}_n\tilde{y}_n \to \tilde{x}\tilde{y}$.

i.e., multiplication in a soft Banach algebra is continuous.

*4) Proposition*

Every parameterized family of crisp Banach algebras on a crisp space V can be considered as a soft Banach algebra on the soft vector space $\widetilde{V}$.

*5) Definition*

A soft element $\tilde{x} \,\tilde{\in}\, \breve{G}$ is said to be invertible if it has an inverse in $\breve{G}$ i.e., if there exists a soft element $\tilde{y} \,\tilde{\in}\, \breve{G}$ such that $\tilde{x}\tilde{y} = \tilde{y}\tilde{x} = \tilde{e}$ and then $\tilde{y}$ is called the invers of $\tilde{x}$, denoted by $\tilde{x}^{-1}$. Otherwise $\tilde{x}$ is said to be non-invertible soft element of $\breve{G}$.

*6) Definition*

A series $\sum_{n=1}^{\infty}\tilde{x}_n$ of soft elements is said to be soft convergent if the partial sum of the series $\tilde{s}_k=\sum_{n=1}^{k}\tilde{x}_n$ is soft convergent.

*7) Theorem*

Let $(G,A)$ be a soft Banach algebra. If $\tilde{x}\tilde{\in}\breve{G}$ satisfies $\|\tilde{x}\|\tilde{<}\bar{1}$, then $(\tilde{e}-\tilde{x})$ is invertible and $(\tilde{e}-\tilde{x})^{-1}=\tilde{e}+\sum_{n=1}^{\infty}\tilde{x}^n$.

*8) Corollary*

Let G be a soft Banach algebra. If $\tilde{x}\tilde{\in}\breve{G}$ and $\|\tilde{e}-\tilde{x}\|\tilde{<}\bar{1}$, then $\tilde{x}^{-1}$ exists and $\tilde{x}^{-1}=\tilde{e}+\sum_{n=1}^{\infty}(\tilde{e}-\tilde{x})^j$.

*9) Corollary*

Let G be a soft Banach algebra. Let $\tilde{x}\tilde{\in}\breve{G}$ and $\tilde{\mu}$ be a soft scaler such that $|\tilde{\mu}|\tilde{>}\|\tilde{x}\|$. Then $(\tilde{\mu}\,\tilde{e}-\tilde{x})^{-1}$ exists and

$$(\tilde{\mu}\,\tilde{e}-\tilde{x})^{-1}=\sum_{n=1}^{\infty}\tilde{\mu}^{-n}\,\tilde{x}_n{}^{n-1};\ (\tilde{x}^o=\tilde{e}).$$

*10) Proposition*

Let G be a soft Banach algebra. The soft set S generated by the set of all invertible soft elements of G is a soft open subset in G.

*11) Definition*

A mapping T from a soft normed linear space G onto G is said to be continuous If for any sequence $\tilde{x}_n$, $\tilde{x}_n\to\tilde{x}$ implies $T(\tilde{x}_n)\to T(\tilde{x})$.

*12) Proposition*

In a soft Banach algebra G, the mapping $\tilde{x}\to\tilde{x}^{-1}$ of S onto S is continuous.

## III. Main Results

Before you begin to format your paper, first write and save the content as a separate text file. Keep your text and graphic files separate until after the text has been formatted and styled. Do not use hard tabs, and limit use of hard returns to only one return at the end of a paragraph. Do not add any kind of pagination anywhere in the paper. Do not number text heads-the template will do that for you.

*1) Definition*

Let $(G,A)$ is a soft Banach algebra. We say that Soft linear functional $\varphi: SE(\breve{G})\to\mathbb{C}(A)$ is soft multiplicative if $\varphi$ is none zero and for $\tilde{x},\tilde{y}\ \tilde{\in}\ \breve{G}$ we have: $\varphi(\tilde{x}\tilde{y})=\varphi(\tilde{x})\,\varphi(\tilde{y})$.

We say that $\varphi$ is soft Jordan multiplicative functional if $\varphi(\tilde{x}^2)=\left(\varphi(\tilde{x})\right)^2$ ; $\forall\tilde{x}\,\tilde{\in}\,\breve{G}$.

*2) Definition*

Let $(G,A)$ be a soft Banach algebra with soft unit element $\tilde{e}$. We denote the soft spectrum of an element $\tilde{x}\ \tilde{\in}\ \breve{G}$ by $Sp(\tilde{x})$ and define it by

$$Sp(\tilde{x})=\{\bar{\lambda}\,\tilde{\in}\,\mathbb{C}(A):\bar{\lambda}\tilde{e}-\tilde{x}\ \tilde{\notin}\ Inv(\breve{G})\},$$

where $Inv(\breve{G})$ is the set of all soft invertible element, of soft banach algebra $(G,A)$. We denote the soft spectral radius of $\tilde{x}$ by $r(\tilde{x})$ and define it by

$$r(\tilde{x})=Sup\{|\bar{\lambda}|:\bar{\lambda}\ \tilde{\in}\ Sp(\tilde{x})\}.$$

*3) Definition*

Let $\bar{o}\,\tilde{<}\,\bar{\varepsilon}\,\tilde{<}\,\bar{1}$. We denote the soft ε_condition spectrum of $\tilde{x}$ in $\breve{G}$ by $Sp_{\bar{\varepsilon}}(\tilde{x})$ and define it by:

$$Sp_{\bar{\varepsilon}}(\tilde{x})=\left\{\bar{\lambda}\ \tilde{\in}\ \mathbb{C}(A):\left\|\left(\bar{\lambda}\tilde{e}-\tilde{x}\right)\right\|\left\|\left(\bar{\lambda}\tilde{e}-\tilde{x}\right)^{-1}\right\|>\frac{\bar{1}}{\bar{\varepsilon}}\right\}.$$

We denote the soft ε_condition spectral radius of $\tilde{x}$ by $r_{\bar{\varepsilon}}(\tilde{x})$ and define it by

$$r_{\bar{\varepsilon}}(\tilde{x})=Sup\{|\bar{\lambda}|:\bar{\lambda}\ \tilde{\in}\ Sp_{\bar{\varepsilon}}(\tilde{x})\}.$$

*4) Lemma*

Let $(G,A)$ be a soft Banach algebra with soft unit element $\tilde{e}$. Let $\tilde{x}\,\tilde{\in}\,\breve{G}$ such that $\|\tilde{x}\|\,\tilde{\leq}\,\bar{1}$ . Then $(\tilde{e}-\tilde{x})$is invertible and $(\tilde{e}-\tilde{x})^{-1}=\tilde{e}+\sum_{n=1}^{\infty}\tilde{x}^n$ . Furthermore we have $\|(\tilde{e}-\tilde{x})^{-1}\|\,\tilde{\leq}\,\frac{\|\tilde{e}\|}{\|\tilde{e}\|-\|\tilde{x}\|}$.

Proof. For the first part see [17] Proposition (4.11). Now let $\tilde{S}_n=\tilde{e}+\tilde{x}+\tilde{x}^2+\cdots+\tilde{x}^n$ and $\tilde{y}=\tilde{e}+\sum_{n=1}^{\infty}\tilde{x}^n$. Then by the first part we know that $(\tilde{e}-\tilde{x})^{-1}=\tilde{y}$. Also we have

$$\|\tilde{y}\|=\lim_{n\to\infty}\|\tilde{S}_n\|=\lim_{n\to\infty}\left\|\tilde{e}+\sum_{k=1}^{n}\tilde{x}^k\right\|\,\tilde{\leq}\,\|\tilde{e}\|+\sum_{k=1}^{\infty}\|\tilde{x}\|^k=\frac{\|\tilde{e}\|}{\|\tilde{e}\|-\|\tilde{x}\|}.$$

If $(G,A)$ is soft unital (i.e., $\|\tilde{e}\|=\bar{1}$) then we have $\|(\tilde{e}-\tilde{x})^{-1}\|\,\tilde{\leq}\,\frac{\bar{1}}{\bar{1}-\|\tilde{x}\|}$.

*5) Corollary*

Let $(G,A)$ be a soft Banach algebra with soft unit element $\tilde{e}$ such that $\|\tilde{e}\|=\bar{1}$ and $\tilde{x}\,\tilde{\in}\,Inv(\breve{G})$. Let $\bar{\lambda}\,\tilde{\in}\,\mathbb{C}(A)-\{\bar{0}\}$ such that $\|\tilde{x}\|\,\tilde{<}\,|\bar{\lambda}|$. Then $(\bar{\lambda}\,\tilde{e}-\tilde{x})$ is invertible and $(\tilde{\mu}\,\tilde{e}-\tilde{x})^{-1}=\sum_{n=1}^{\infty}\tilde{\mu}^{-n}\,\tilde{x}_n{}^{n-1}$ ; $(\tilde{x}^o=\tilde{e})$. Furthermore we have

$$\left\|\left(\bar{\lambda}\tilde{e}-\tilde{x}\right)^{-1}\right\|\,\tilde{\leq}\,\frac{\bar{1}}{|\bar{\lambda}|-\|\tilde{x}\|}.$$

Proof. For the first part see[17] corollary (4.13). To prove the last statement we need only to substitute $\frac{\tilde{x}}{\bar{\lambda}}$ with $\tilde{x}$ in Lemma (3.4) and we get the result.

*6) Theorem*

Let $(G,A)$ be a soft Banach algebra with soft unit element $\tilde{e}$ such that $\|\tilde{e}\|=\bar{1}$ and $\tilde{x}\,\tilde{\in}\,\breve{G}$ . Then we have $r(\tilde{x})\,\tilde{\leq}\,r_{\bar{\varepsilon}}(\tilde{x})\,\tilde{\leq}\,\frac{\bar{1}+\bar{\varepsilon}}{\bar{1}-\bar{\varepsilon}}\|\tilde{x}\|$.

Proof. Since $Sp(\tilde{x})\,\tilde{\subseteq}\,Sp_{\bar{\varepsilon}}(\tilde{x})$, so we have $r(\tilde{x})\,\tilde{\leq}\,r_{\bar{\varepsilon}}(\tilde{x})$. Suppose that $\bar{\lambda}\,\tilde{\in}\,Sp_{\bar{\varepsilon}}(\tilde{x})$. If $|\bar{\lambda}|\,\tilde{\leq}\,\|\tilde{x}\|$, then we can easily prove that $|\bar{\lambda}|\,\tilde{\leq}\,\frac{\bar{1}+\bar{\varepsilon}}{\bar{1}-\bar{\varepsilon}}\|\tilde{x}\|$. Thus we have $r_{\bar{\varepsilon}}(\tilde{x})\,\tilde{\leq}\,\frac{\bar{1}+\bar{\varepsilon}}{\bar{1}-\bar{\varepsilon}}\|\tilde{x}\|$.

Now suppose that $|\bar{\lambda}|\,\tilde{>}\,\|\tilde{x}\|$. Then $(\bar{\lambda}\tilde{e}-\tilde{x})$ is invertible and by corollary (3.5) we have $\left\|\left(\bar{\lambda}\tilde{e}-\tilde{x}\right)^{-1}\right\|\,\tilde{\leq}\,\frac{\bar{1}}{|\bar{\lambda}|-\|\tilde{x}\|}$. Thus we obtain

$$\bar{1}\,\tilde{\leq}\,\left\|\left(\bar{\lambda}\tilde{e}-\tilde{x}\right)^{-1}\right\|\left\|\left(\bar{\lambda}\tilde{e}-\tilde{x}\right)\right\|\,\tilde{\leq}\,\bar{\varepsilon}\left(\frac{|\bar{\lambda}|+\|\tilde{x}\|}{|\bar{\lambda}|-\|\tilde{x}\|}\right).$$

Consequently by some computations we get $|\bar{\lambda}| \precsim \frac{\bar{1}+\bar{\varepsilon}}{\bar{1}-\bar{\varepsilon}}\|\tilde{x}\|$. Thus we conclude that $r_{\bar{\varepsilon}}(\tilde{x}) \precsim \frac{\bar{1}+\bar{\varepsilon}}{\bar{1}-\bar{\varepsilon}}\|\tilde{x}\|$.

*7) Definition*

Let $(G, A)$ be a soft Banach algebra and $T: SE(\breve{G}) \to \mathbb{C}(A)$ be a soft linear functional. We say that $T$ is almost soft multiplicative if there exists an $\bar{\delta} \succsim \bar{o}$ such that for all $\tilde{x}, \tilde{y} \,\tilde{\in}\, \breve{G}_1$:

$$|T(\tilde{x}\,\tilde{y}) - T(\tilde{x})T(\tilde{y})| \precsim \bar{\delta}\|\tilde{x}\|\|\tilde{y}\|.$$

*8) Proposition*

Let $\varphi$ is a soft linear functional on a soft Banach algebra $(G, A)$ with unit element $\tilde{e}$ such that $\varphi(\tilde{e}) = \bar{1}$. Then the following conditions are equivalent.

i) $\varphi(\tilde{x}) = \bar{0}$ implies $\varphi(\tilde{x}^2) = \bar{0}$ for all $\tilde{x} \,\tilde{\in}\, \breve{G}$,

ii) $\varphi(\tilde{x}^2) = \left(\varphi(\tilde{x})\right)^2$, $\tilde{x} \,\tilde{\in}\, \breve{G}$,

iii) $\varphi(\tilde{x}) = \bar{0}$ implies $\varphi(\tilde{x}\tilde{y}) = \bar{0}$ for all $\tilde{x}, \tilde{y} \,\tilde{\in}\, \breve{G}$,

iv) $\varphi(\tilde{x}\tilde{y}) = \varphi(\tilde{x})\,\varphi(\tilde{y})$ $\forall \tilde{x}, \tilde{y} \,\tilde{\in}\, \breve{G}$.

Proof : (i) $\Rightarrow$ (ii)

$\varphi(\tilde{e}) = \bar{1}$ implies

$$\varphi(\tilde{x} - \varphi(\tilde{x}))(\lambda) = \varphi(\tilde{e}\tilde{x} - \varphi(\tilde{x}))(\lambda) = \varphi(\tilde{e})(\lambda)\,\varphi(\tilde{x})(\lambda) - \varphi(\tilde{x})(\lambda) = 0 \;;\; \forall \lambda \in A.$$

So we have $\bar{0} = \varphi(\tilde{x} - \varphi(\tilde{x})) = \bar{0}$. By (i) we have

$$\bar{0} = \varphi\left(\left(\tilde{x} - \varphi(\tilde{x})\right)^2\right) = \varphi\left(\tilde{x}^2 - \bar{2}\tilde{x}\,\varphi(\tilde{x}) + \left(\varphi(\tilde{x})\right)^2\right) = \varphi(\tilde{x}^2) - \left(\varphi(\tilde{x})\right)^2.$$

Thus we deduce that $\varphi(\tilde{x}^2) = \left(\varphi(\tilde{x})\right)^2$.

(ii) $\Rightarrow$ (iii)

By replacing $\tilde{u} + \tilde{v}$ with $(\tilde{x})$ in (ii) we get $\varphi(\tilde{u}\tilde{v} + \tilde{v}\tilde{u}) = \bar{2}\varphi(\tilde{u})\varphi(\tilde{v})$ ; $\tilde{u}, \tilde{v} \in \breve{G}$, (1)

Let $\tilde{x}, \tilde{y}$ be in $\breve{G}$ with $\varphi(\tilde{x}) = \bar{0}$. According to (1) we have $\varphi(\tilde{x}\tilde{y} + \tilde{y}\tilde{x}) = \bar{0}$. (2)

Hence by (ii) we obtain $\varphi((\tilde{x}\tilde{y} + \tilde{y}\tilde{x})^2) = \bar{0}$. Since

$$(\tilde{x}\tilde{y} - \tilde{y}\tilde{x})^2 = \bar{2}\,\varphi(\tilde{x}(\tilde{y}\tilde{x}\tilde{y}) + (\tilde{y}\tilde{x}\tilde{y})\tilde{x}) = \bar{4}\,\varphi(\tilde{x})\,\varphi(\tilde{x}\tilde{y}\tilde{x}) = \bar{0}.$$

According to (ii) we have $\varphi(\tilde{x}\tilde{y} - \tilde{y}\tilde{x}) = \bar{0}$, (3). If we add two equalities (2) and (3) we conclude that $\varphi(\tilde{x}\tilde{y}) = \bar{0}$.

(iii) $\Rightarrow$ (iv)

Let $\tilde{x}, \tilde{y} \,\tilde{\in}\, \breve{G}$. We have $\varphi(\tilde{x} - \varphi(\tilde{x})) = \bar{0}$. Hence for each $\lambda \in A$ we have

$\varphi(\tilde{x} - \varphi(\tilde{x}))(\lambda) = 0$. Thus by (iii) we have

$$\varphi\left(\left(\tilde{x} - \varphi(\tilde{x})\right)\tilde{y}\right)(\lambda) = \bar{0}.$$

Then we get

$$0 = \varphi\left(\left(\tilde{x} - \varphi(\tilde{x})\right)\tilde{y}\right)(\lambda) = \varphi(\tilde{x}\tilde{y} - \varphi(\tilde{x})\tilde{y})(\lambda) = \varphi(\tilde{x}\tilde{y})(\lambda) - \varphi(\tilde{x})(\lambda)\,\varphi(\tilde{y})(\lambda) \;;\quad \forall \lambda \in A.$$

Thus we have $\bar{0} = \varphi(\tilde{x}\tilde{y}) - \varphi(\tilde{x})\varphi(\tilde{y})$. Consequently we get $\varphi(\tilde{x}\tilde{y}) = \varphi(\tilde{x})\varphi(\tilde{y})$.

(iv) $\Rightarrow$ (i)

From (iv) we have $\varphi(\tilde{x}\tilde{y})(\lambda) = \varphi(\tilde{x})(\lambda)\,\varphi(\tilde{y})(\lambda)$. If $\varphi(\tilde{x}) = \bar{0}$, then $\varphi(\tilde{x})(\lambda) = 0$; $\forall \lambda \in A$. So we have

$$\varphi(\tilde{x}\tilde{y})(\lambda) = \varphi(\tilde{x})(\lambda)\,\varphi(\tilde{y})(\lambda) = 0\,.\,\varphi(\tilde{y})(\lambda) = 0.$$

Therefore $\varphi(\tilde{x}\tilde{y}) = \bar{0}$.

Clearly if $\varphi$ is soft multiplicative linear functional then $\varphi$ is soft Jordan multiplicative functional. Now we have the following corollary.

*9) Corollary*

Let $\varphi$ is a soft Jordan multiplicative linear functional on soft Banach algebra $(G, A)$ with unit element $\tilde{e}$ such that $\varphi(\tilde{e}) = \bar{1}$. Then $\varphi$ is soft multiplicative.

*10) Lemma*

Let $\varphi$ is a soft multiplicative linear functional on soft Banach algebra $\breve{G}$.

Then we have $\varphi(\tilde{x}) \,\tilde{\in}\, sp(\tilde{x})$ ; $\tilde{x} \,\tilde{\in}\, \breve{G}$.

Proof: For $\tilde{x} \,\tilde{\in}\, \breve{G}$ we set $\tilde{z} = \varphi(\tilde{x})\tilde{e} - \tilde{x}$. Then we have

$$\varphi(\tilde{z})(\lambda) = \varphi(\tilde{x})(\lambda)\,\varphi(\tilde{e})(\lambda) - \varphi(\tilde{x})(\lambda) = \varphi(\tilde{x})(\lambda) - \varphi(\tilde{x})(\lambda) = 0.$$

Hence we have $\varphi(\tilde{z}) = \bar{0}$. Therefore $\tilde{z} \,\tilde{\in}\, \ker\varphi$. So we have $\tilde{z} \,\tilde{\in}\, sing(\breve{G})$. Consequently $\varphi(\tilde{x}) \,\tilde{\in}\, sp(\tilde{x})$.

*11) Remark*

Now suppose that $\varphi$ is soft multiplicative linear functional and let $\tilde{z} \,\tilde{\in}\, sp(\tilde{x})$, for some $\tilde{x} \,\tilde{\in}\, \breve{G}$. Then $\tilde{z}\tilde{e} - \tilde{x}$ is not invertible and so we have $\varphi(\tilde{z}\tilde{e} - \tilde{x}) = \bar{0}$.

We know that, for invertible element $\tilde{y}$ we have $\varphi(\tilde{y}) = \bar{0}$. Thus $\tilde{z}\varphi(\tilde{e}) - \varphi(\tilde{x}) = \bar{0}$. So we have $\tilde{z} = \varphi(\tilde{x})$. Consequently we have the following theorem.

*12) Theorem*

Let $(G, A)$ be a commutative soft Banach algebra and let $\tilde{x} \,\tilde{\in}\, \breve{G}$, then

$$Sp(\tilde{x}) = \{\varphi(\tilde{x}) : \varphi \text{ is a soft multiplicative linear functional}\}.$$

Proof. It follows from lemma (3.10) and last statement.

*13) Lemma*

Let $T: SE(\breve{G}_1) \to \mathbb{C}(A)$ be a soft bounded linear functional. Then $T$ is almost soft multiplicative.

Proof: For each $\tilde{x}, \tilde{y} \,\tilde{\in}\, \breve{G}_1$ we have

$$|T(\tilde{x}\,\tilde{y}) - T(\tilde{x})T(\tilde{y})| \leq |T(\tilde{x}\,\tilde{y})| + |T(\tilde{x})T(\tilde{y})|$$

$$\leq \|T\|\|\tilde{x}\,\tilde{y}\| + \|T\|^2\|\tilde{x}\|\|\tilde{y}\| = (\|T\| + \|T\|^2)\|\tilde{x}\|\|\tilde{y}\|.$$

Thus $T$ is almost soft multiplicative where $\bar{\delta} = (\|T\| + \|T\|^2)$.

*14) Proposition*

Let $(G, A)$ is a soft Banach algebra and $T_1: SE(\breve{G}_1) \rightarrow \mathbb{C}(A)$ is a soft multipicactive linear functional and $T_2: SE(\breve{G}_1) \rightarrow \mathbb{C}(A)$ is a soft bounded linear functional. Then $T_1 + T_2$ is almost soft multiplicative functional but not multiplicative.

Proof: For each $\tilde{x}, \tilde{y} \tilde{\in} \breve{G}$ we have

$$|(T_1 + T_2)(\tilde{x}\,\tilde{y}) - (T_1 + T_2)(\tilde{x})(T_1 + T_2)(\tilde{y})|$$

$$= \left|T_1(\tilde{x}\,\tilde{y}) + T_2(\tilde{x}\,\tilde{y}) - \left(T_1(\tilde{x}) + T_2(\tilde{x})\right)\left(T_1(\tilde{y}) + T_2(\tilde{y})\right)\right| = |T_1(\tilde{x}\,\tilde{y}) + T_2(\tilde{x}\,\tilde{y}) - T_1(\tilde{x})T_1(\tilde{y}) - T_2(\tilde{x})T_2(\tilde{y}) - T_1(\tilde{x})T_2(\tilde{y}) - T_2(\tilde{x})T_1(\tilde{y})|$$

$$\leq |T_1(\tilde{x}\,\tilde{y}) - T_1(\tilde{x})T_1(\tilde{y})| + |T_2(\tilde{x}\,\tilde{y}) - T_2(\tilde{x})T_2(\tilde{y})| + |T_1(\tilde{x})T_2(\tilde{y})| + |T_2(\tilde{x})T_1(\tilde{y})|.$$

So by lemma (3.11) we get

$$|(T_1 + T_2)(\tilde{x}\,\tilde{y}) - (T_1 + T_2)(\tilde{x})(T_1 + T_2)(\tilde{y})|$$

$$\leq (\|T_2\| + \|T_2\|^2)\|\tilde{x}\|\|\tilde{y}\| + \bar{2}\|T_1\|\|\tilde{x}\|\|T_2\|\|\tilde{y}\|$$

$$= (\|T_2\| + \|T_2\|^2 + \bar{2}\|T_1\|\|T_2\|)\|\tilde{x}\|\|\tilde{y}\|.$$

Thus $(T_1 + T_2)$ is almost soft multiplicative. Clearly $(T_1 + T_2)$ is not multiplicative.

*15) Definition*

Let $(G, A)$ be a soft Banach algebra. We say that soft linear functional $\varphi: SE(\breve{G}) \rightarrow \mathbb{C}(A)$ is almost soft Jordan multiplicative functional if there exist $\bar{\delta} \tilde{>} \bar{o}$ such that:

$$|\varphi(\tilde{x}^2) - \varphi(\tilde{x})^2| \tilde{\leq} \bar{\delta}\|\tilde{x}\|^2, \quad \forall \tilde{x} \tilde{\in} \breve{G}.$$

*16) Corollary*

Let $(G, A)$ be a soft Banach algebra and $T_1: SE(\breve{G}) \rightarrow \mathbb{C}(A)$ is a soft Jordan multiplicative linear functional and $T_2: SE(\breve{G}) \rightarrow \mathbb{C}(A)$ is a soft bounded linear functional. Then $T_1 + T_2$ is almost soft Jordan multiplicative linear functional.

Proof: It can be proved by similar method which we stated in theorem(3.12).

*17) Definition*

Let $(G, A)$ be a soft Banach algebra with unit element $\tilde{e}$ and let $\bar{\varepsilon} \tilde{>} \bar{o}$. We denote the soft ε_condition spectrum of an element $\tilde{x} \tilde{\in} \breve{G}$ by $\Lambda_{\bar{\varepsilon}}(\tilde{x})$ and define it by:

$$\Lambda_{\bar{\varepsilon}} = \left\{\bar{\lambda} \tilde{\in} \mathbb{C}(A): \left\|\left(\bar{\lambda}\tilde{e} - \tilde{x}\right)^{-1}\right\| \tilde{\geq} \frac{\bar{1}}{\bar{\varepsilon}}\right\}.$$

*18) Theorem*

Let $(G, A)$ be a soft Banach algebra with unit element $\tilde{e}$ and let $\bar{\varepsilon} \tilde{>} \bar{o}$. Let $\varphi: SE(\breve{G}) \rightarrow \mathbb{C}(A)$ be a soft linear functional such that $\varphi(\tilde{e}) = \bar{1}$ and $\varphi(\tilde{e}) \tilde{\in} \Lambda_{\bar{\varepsilon}}(\tilde{x})$ for $\tilde{x} \tilde{\in} \breve{G}$. Then $\varphi$ is soft multiplicative functional.

Proof: We prove that for all $\tilde{x} \tilde{\in} \breve{G}$ we have $\varphi(\tilde{x}) \tilde{\in} Sp(\tilde{x})$. We put $\bar{\lambda} = \varphi(\tilde{x})$. If $\bar{\lambda} \tilde{\in} Sp(\tilde{x})$ then $\varphi$ is multiplicative. If $\bar{\lambda} \tilde{\notin} Sp(\tilde{x})$ then $\bar{\lambda}\tilde{e} - \tilde{x}$ is invertible and so $\bar{\lambda}\tilde{e} - \tilde{x} \tilde{\in} Inv(\breve{G})$.

Suppose that $\bar{z} \tilde{>} \bar{\varepsilon}\left\|\left(\bar{\lambda}\tilde{e} - \tilde{x}\right)^{-1}\right\|$.

Then we have $\left\|\left(\bar{\lambda}\tilde{e} - \tilde{x}\right)^{-1}\right\| \tilde{<} \frac{\bar{1}}{\bar{\varepsilon}}$.

Thus we get $\left\|\left(\bar{\lambda}\tilde{e}\bar{z} - \tilde{x}\bar{z}\right)^{-1}\right\| \tilde{<} \frac{\bar{1}}{\bar{\varepsilon}}$

Consequently we have $\bar{\lambda}\bar{z} = \varphi(\tilde{x})\bar{z} = \varphi(\bar{z}\tilde{x}) \notin \Lambda_{\bar{\varepsilon}}(z\bar{x})$, which is a contradiction. So $\varphi$ is soft multiplicative.

*19) Lemma. Let $\bar{\delta} \tilde{>} \bar{o}$ and $\tilde{x} \tilde{\in} \breve{G}$. Then $Sp(\tilde{x}) \subseteq Sp_{\bar{\delta}}(\tilde{x})$.*

Proof: It can be proved easily by definition.

*20) Theorem*

Let $(G, A)$ be a soft Banach algebra with unit element $\tilde{e}$ and $\varphi$ be an almost soft multiplicative linear functional on $\breve{G}$. If $\varphi(\tilde{e}) = \bar{1}$. Then for every element $\tilde{x} \tilde{\in} \breve{G}$ we have $\varphi(\tilde{x}) \tilde{\in} Sp_{\bar{\delta}}(\tilde{x})$.

Proof: Let $\tilde{x} \tilde{\in} \breve{G}$ and $\bar{\lambda} = \varphi(\tilde{x})$. If $\bar{\lambda}\tilde{e} - \tilde{x}$ is not invertible then $\bar{\lambda} \tilde{\in} Sp(\tilde{x}) \subseteq Sp_{\bar{\delta}}(\tilde{x})$. So $\bar{\lambda} \tilde{\in} \sigma_{\bar{\delta}}(\tilde{x})$. Now assume that $\bar{\lambda}\tilde{e} - \tilde{x}$ is invertible. Then

$$\bar{1} = |\varphi(\tilde{e})| = |\varphi(\tilde{x}) - \bar{o}\,| =$$

$$\left|\varphi(\tilde{e}) - \varphi\left(\bar{\lambda}\tilde{e} - \bar{x}\right)\varphi\left(\left(\bar{\lambda}\tilde{e} - \bar{x}\right)^{-1}\right)\right|$$

$$\tilde{\leq} \bar{\delta}\left\|\left(\bar{\lambda}\tilde{e} - \bar{x}\right)\left(\left(\bar{\lambda}\tilde{e} - \bar{x}\right)^{-1}\right)\right\|$$

Thus we have

$$\left\|\left(\bar{\lambda}\tilde{e} - \bar{x}\right)\left(\left(\bar{\lambda}\tilde{e} - \bar{x}\right)^{-1}\right)\right\| \tilde{\geq} \frac{\bar{1}}{\bar{\delta}}.$$

So we conclude that $\bar{\lambda} \tilde{\in} Sp_{\bar{\delta}}(\tilde{x})$. Consequently we have $\varphi(\tilde{x}) \tilde{\in} Sp_{\bar{\delta}}(\tilde{x})$.

## IV. CONCLUSION

In this paper we introduced the ideas of soft spectrum, soft condition spectrum, soft ε_condition spectrum and soft spectral radius of a soft element over soft Banach algebras. We studied some basic properties of these ideas in soft Banach algebras. Also we defined the notions of soft multiplicative linear functional, almost soft multiplicative linear functional, soft Jordan multiplicative linear functional and almost soft Jordan multiplicative linear functional in soft Banach algebras. Finally some new results and theorems about them over soft Banach algebra have investigated.

## REFERENCES


[1] H. Aktas and N. Cagman, Soft sets and soft groups, Inform. Sci. 177 (2007) 2226-2735.

[2] N. Cagman, S. Karatas and S. Enginoglu, Soft topology, Comput. Math. Appl. 62 (2011) 351-358.

[3] S. Das, P. Majumdar and S. K. Samanta, On Soft linear spaces and soft normed linear spaces, Ann. Fuzzy Math Inform. 9 (1) (2015) 91-109.

[4] Das and S. K. Samanta, Soft Real Sets, Soft Real Numbers and Their Properties, J. Fuzzy Math. 20 (3) (2012) 551-576.

[5] S. Das and S. K. Samanta, On Soft Complex Sets and Soft Complex Numbers, J. Fuzzy Math. 21 (1) (2013) 195-216.

[6] S. Das and S. K. Samanta, On Soft Metric Spaces, J. Fuzzy Math. 21(3) (2013) 707-734.

[7] S. Das, P. Majumdar and S. K. Samanta, On Soft Linear Spaces and Soft Normed Linear Spaces, Ann. Fuzzy Math. Inform. 9 (1) (2015) 91-109.

[8] S. Das and S.K. Samanta, On soft inner product spaces, Ann. Fuzzy Math. Inform. 6 (1) (2013) 151-170.

[9] S. Das and S. K. Samanta, Soft linear operators in soft normed linear spaces, Ann. Fuzzy Math. Inform. 6 (2) (2013) 295-314.

[10] S. Das and S. K. Samanta, Soft linear functionals in soft normed linear

[11] S. Das and S. K. Samanta, Operators on soft inner product spaces, Fuzzy Inf. Eng. 6 (2014) 435-450.

[12] P. K. Maji, R. Biswas and A. R. Roy, An application of soft sets in a decision making problem,Comput. Math. Appl. 44 (2002) 1077-1083.

[13] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555-562.

[14] D. Molodtsov, Soft set theory first results, Comput. Math. Appl. 37 (1999) 19-31.

[15] S. H. J. Petroudi, S. A. Sadati, Z. Nabati, From soft set theory to a decision making problem, The first conference on modeling mathematics and statistics in applied studies. (2017), Chalos, Iran.

[16] S. H. J. Petroudi, S. A. Sadati, A. Yaghobi, Almost Soft multiplicative linear functional, The first national conference on Applied mathematics in engineering and basic science. (2017), Shiraz, Iran.

[17] R. Thakur, S. K. Samanta, Soft Banach Algebra, Ann. Fuzzy Math. Inform. 10 (3) (2015) 397-412.