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Estimating Hoek-Brown Criterion Parameters for Coarse-Grained-Soil-Cement Mixtures

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Abstract- Properties of soil-cement mixtures have been studied extensively. However, how to determine the parameters for Hoek-Brown (H-B) criterion for soil-cement mixtures from these properties remains unsolved. This study proposes an optimization-based method to obtain the fitting parameters for Hoek-Brown criterion from the cohesion and the friction angle of coarse-grained-soil-cement mixtures, details of the procedure to optimize the fitting parameters are presented. A numerical investigation using the H-B criterion with the parameters obtained from the proposed procedure for coarse-gained-soil-cement mixtures is conducted for a case study, influences of fitting parameters on the calculated settlement are studied and comparisons with that using M-C criterion and field monitored data are present.

Keywords- *Hoek-Brown Criterion, Mohr-Coulomb Failure Criterion, Coarse-Grained-Soil-Cement Mixtures, Ground Excavation,Ssurface Subsidence*

I. INTRODUCTION

Grouting is a widely used method for stabilizing soil and providing support for ground excavations, particularly for those constructed in soft ground or requiring irregular geometry of cross-sections (e.g., subway transfer stations). Grouting is the general term for the technique whereby cementitious chemical materials are injected into the soil pores, which occur either naturally or are man-made, through various mechanical actions (ASCE 1995). As shown in Fig. 1, the grout material, usually cement, penetrates into the native ground and result in a concrete- or rock-like material that exhibits a significant increase in the shear strength. The final product is best described as fine- or coarse-grained-soil-cement mixtures in terms of the particle size of native soils, and this term will be used herein.

When grouting cement to ground to form a stabilized block of soil-cement mixtures, the specifications often focus on an unconfined compressive strength q_u, which is typically in the range of 1.5 to 2.5 MPa, rather than a required Mohr-Coulomb strength (Wang et al. 2015, Yoo and Shin 2003). In the numerical analysis to calculate the excavation-induced deformations, the soil-cement mixed volumes are usually treated as a soil-like material (e.g., Bae et al. 2005, Yoo 2002, Yoo and Shin 2003), and the Mohr-Coulomb (M-C) failure criterion is usually adopted. However, some researchers have pointed out that M-C is not suitable for excavation-induced settlement analysis (Mollon et al. 2013), especially a material with non-linear shear strength (Barton 2013). Moreover, core samples of soil-cement mixtures are sometimes generally similar to weak rock or lean concrete, as shown in Fig. 1. Compared with Terzaghi's (1946) rock classification, the artificial soil-cement mixture may be more appropriate to be considered as blocky and seamy rock consisting of chemically intact or almost intact rock fragments, and thus the Hoek-Brown (H-B) criterion might be an alternative. However, how to determine the parameters for H-B criterion for grouting-stabilized excavation from the easily-obtained properties of soil-cement mixtures remains unsolved.

Figure 1. Core samples of soil-cement mixtures

Aiming to obtain H-B criterion parameters more conveniently from the c-ϕ values, this paper proposes an Excel-based optimization approach to derive the H-B criterion parameters for soil-cement mixtures. Analytical solutions, which were developed by Kumar (1998) originally and further developed by Shen et al. (2012) to determine the c-ϕ parameters from the Hoek-Brown criterion parameters, are rearranged based on graphical expressions of the two criteria, an optimization procedure using Solver in Excel® is proposed to obtain the parameters of m_b, s and a of H-B criterion in addition to GSI obtained from the core samples of soil-cement mixtures. A numerical investigation using the obtained parameters is carried out for a stabilized excavation project, the influence of parameters m_b and s on the analysis result is studied and comparisons with the results using M-C criteria and field monitored data is presented.

II. Obtaining the Hoek-Brown Criterion Fitting Parameters from *c*-ϕ Values

A. *Mohr-Coulomb and Hoek-Brown Criteria*

Depending on drainage conditions, the M-C failure criterion uses either effective or total-stress strength parameters, the cohesion *c* and the internal friction angle ϕ, to describe the shear stress at failure τ_{MC}. M-C criterion is expressed by (effective stress is cited herein):

$$\tau'_{MC} = c' + \sigma'_{n\_MC} \tan \phi' \tag{1}$$

where σ'_{n_MC} is the effective normal stress at failure on the failure plane following Mohr-Coulomb failure criterion. The yield surface (i.e., the boundary of the elastic region) is defined by (Labuz and Zang 2012, Want et al. 2015):

$$F(\{\sigma'\},\{k\}) = \sigma'_1 - \sigma'_3 - 2c' \cos \phi' - (\sigma'_1 + \sigma'_3) \sin \phi' \tag{2}$$

where $F(\{\sigma'\},\{k\})$ is the yield function, σ'_1 and σ'_3 are the major and minor principal stresses, {k} is the matrix of material constants. On the other hand, the Hoek–Brown failure criterion is an empirically derived relationship used to describe a non-linear increase in peak strength of isotropic rock with increasing confining stress, which has since been modified by Hoek and Brown (1988), and Hoek et al. (2002, 2013). The generalized Hoek-Brown criterion is expressed as

$$\sigma'_1 = \sigma'_3 + \sigma_{ci}(m_b \sigma'_3 / \sigma_{ci} + s)^a \tag{3}$$

where σ_{ci} is the uniaxial compressive strength of the intact rock, which is essentially equal to the unconfined compressive strength q_u for soil-cement mixtures. m_b, *s* and *a* are the fitting parameters of the Hoek-Brown criterion. These fitting parameters of H-B criterion are given by (Hoek et al. 2002):

$$m_b = m_i e^{(GSI-100)/(28-14D)} \tag{4}$$

$$s = e^{(GSI-100)/(9-3D)} \tag{5}$$

$$a = 0.5 + (e^{-GSI/15} - e^{-20/3})/6 \tag{6}$$

where m_i is the Hoek-Brown constant for intact rock mass, *D* is the disturbance factor which depends - on the degree of disturbance to which the rock mass has been subjected to the blast damage and stress relaxation. Separately, and of interest for deformation analyses, the modulus of deformation for rock masses, E_m, for cases when $\sigma_{ci} \le 100$ MPa may be estimated using the GSI and disturbance factor (Hoek et al. 2002):

$$E_m(GPa) = (1 - D/2)\sqrt{\sigma_{ci}/100}\,(10^{((GSI-10)/40)}) \tag{7}$$

III. An Optimization-Based Approach to Obtain the H-B Criterion Fitting Parameters

Fig. 2 shows a comparison of the M-C and H-B failure criteria for *c*-ϕ materials, which indicates that the two failure envelopes intersect at point A. In addition, considering the H-B envelope is non-linear, it may intersect with the M-C envelope at another point B (Eberhardt 2012). The graph is used as the basis for the derivation of the Hoek-Brown criterion parameters from M-C parameters conducted herein.

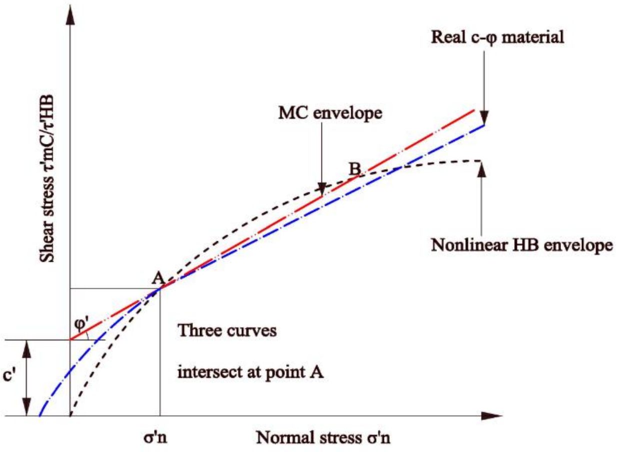

Figure 2. Failure envelopes of M-C and H-B criterion

Kumar (1998) originally developed and Shen el. (2012) cited and further developed a shear failure envelope of the Hoek-Brown criterion for rock masses. The equations can be rearranged as follows:

$$\sigma'_{n\_HB} = \frac{\sigma_{ci}}{m_b}\left[\left[\left(\frac{m_b a}{2}\right)^{\left(\frac{1}{1-a}\right)}\left(\frac{1-\sin\phi'}{\sin\phi'}\right)^{\left(\frac{1}{1-a}\right)}\right)\left(1+\frac{\sin\phi'}{a}\right)\right] - s\right] \tag{8}$$

$$\tau'_{HB} = \frac{\sigma_{ci} \cos\phi'}{2(1+\sin\phi'/a)^a}\left(m_b \frac{\sigma'_n}{\sigma_{ci}} + s\right)^a \tag{9}$$

where σ'_{n_HB} and τ'_{HB} are effective normal and shear stress at the failure plane for H-B criterion, respectively. Note that Kumar's original paper used instantaneous friction angle β instead of friction angle ϕ' in Equations (8-9). The instantaneous friction angle was proposed by Barton and Bandis (1982) to address the variation of shear stress and normal stress and defined $\beta = \arctan(\partial\tau/\partial\sigma_n)$. However, comparing the internal friction angle ϕ' in soil mechanics with the illustration for the instantaneous angle β in Kumar's paper indicate that they are essentially the same, and Shen et al. (2012) use the angle of friction ϕ' directly. It will be assumed that σ'_{n_HB} and τ'_{HB} satisfies the Mohr-Coulomb criterion expressed by Equation (1), i.e., the state of stress is represented by the conditions at point A (Fig. 2). The unconfined compressive strength, q_u, is the designed and assumed known parameter, the corresponding effective cohesion, and the effective friction angle of the soil-cement mixture can be estimated based on the q_u (Balmer 1958, Wang et al. 2014) and a GSI for soil-cement mixture can be estimated based on RQD for the core samples. Thus three unknown fitting parameters, m_b , *s* and *a*, for the H-B criterion can be solved theoretically based on Equations (3) (8) and (9). Practically, numerical optimization using Solver® in Excel® by setting the optimization target $\left|(\tau'_{HB}-\tau'_{MC})\right|+\left|\sigma'_{n\_HB}-\sigma'_{n\_MC}\right|$ equal zero which means the shear strengths from the two criteria are equal, can be carried out to obtain the solutions.

IV. OPTIMIZATION PROCESS

Realizing the challenges in estimating the *GSI* by engineers, Hoek et al. (2013) developed a method to quantify the *GSI* for numerical analysis. The estimated GSI can be calculated by:

$$GSI = 1.5JCond_{89} + RQD/2 \tag{10}$$

To use their method to estimate *GSI*, the Rock Quality Designation (RQD) index of core samples of soil-cement mixtures should be measured in addition to estimating the rating of discontinuity conditions of the soil-cement mixture by comparing the core samples with the description of the rating system for discontinuity, as described in Table 1 (Bieniawski 1989, Hoek et al. 2013). For illustration purposes, based on an RQD of 60% (using field measurement) and $JCON_{89}$ =10 (using Table 1with field judgment) for the core sample of soil-cement mixture as illustrated in Fig. 1, a GSI=45 can be obtained using Equation (10). Previous studies showed that the internal angle of friction is approximate 43º for coarse-grained-soil-cement mixtures (Balmer 1958, Wang et al. 2014b). As for cohesion c', the following Equation was proposed by Mitchell (1976) and used by Wang et al (2014):

$$c' = 48.265 + 0.2251\, q_u \tag{11}$$

where q_u is the unconfined compressive strength at 28 days of soil-cement mixture in units of kPa, and c' is also in a unit of kPa. Equation (11) can be used in Table 2 to obtain the cohesion c'.

TABLE I. Definition of JCon89 after Bieniawski (1989) and Hoek et al.(2013)

Condition of discontinuities	Very rough surface, Not continuous, No separation, Unweathered rock	Slightly rough surface, Separation<1 mm, Slightly weathered walls	Slightly rough surface, Separation<1 mm, Highly weathered walls	Slickensided surface or Gouge<5 mm thick or Separation 1-5 mm continues	Soft gouge > 5 mm thick or Separation> 5 mm continues
Rating	30	25	20	10	0
Guidelines for classification of discontinuity conditions					
Discontinuity length (persistence) Rating	<1 m 6	1 to 3 m 4	3 to 10 m 2	10 to 20 m 1	More than 20 m 0
Separation (aperture) Rating	None 6	<0.1 mm 5	0.1-1.0 mm 4	1-5 mm 1	>5 mm 0
Roughness Rating	Very rough 6	Rough 5	Slightly rough 3	Smooth 1	Slickensided 0
Infilling (gouge) Rating	None 6	Hard filling < 5 mm 4	Hard filling >5 mm 2	Soft filling < 5 mm 2	Soft filling > 5 mm 0
Weathering Rating	Unweathered 6	Slightly weathered 5	Moderate weathering 3	Highly weathered 2	Decomposed 0

Table 2 was originally generated using Excel ® based on above analysis, but in this paper Table 2 was presented in word format. The first part of Table 2 is "input area", where the strength parameters of M-C criteria and the estimated GSI can be manually placed into the cells. The second part in Table 2 is for "optimizing H-B fitting parameters", i.e., m_b, *s* and *a* values will be optimized. m_b is an adjusted parameter related to m_i which is for intact rock and usually is determined by triaxial tests. Instead of running triaxial tests to obtain m_i, an estimated number of m_b say 5 can be used as a starting point for optimization while *s* and *a* can be obtained by using Equations (5-6). Nevertheless, constraints $0 \le m_b \le 35$, $0 \le s \le 1$ and $0.5 \le a \le 0.666$ are added to m_b , *s* and *a* respectively to prevent yielding unrealistic results. The third part of Table 2 is "Optimization target", where (τ_{MC}-τ_{HB})=0 and (σ_{n_HB}-q_u)=0 are intermediate process, the optimizing target is $|\sigma_{n\_HB}$-$q_u|+|\tau_{MC}$-$\tau_{HB}|$=0. Solver® shall target this cell to optimize its values by changing the values of cells for m_b, *s* and *a*.

During the process, m_b increases to above 11 after running the optimization, while *s* and *a* are also optimized.

Unfortunately, Solver® in Excel® cannot fully automatically optimize the solutions to reach the optimization target in accordance with Fig. 2 and Equations (8-9), the starting values of m_b and *a* need to be changed manually after each iteration. The optimization target also needs to be recorded and compared for all iterations.

To understand the process better, the flowchart of the procedure is described in Fig. 3. When the optimization target reached, the optimized H-B fitting parameters, m_b , *s* and *a* can be used for numerical analysis.

TABLE II. OPTIMIZED PARMETERS OF H-B CRITERION FOR COARSE-GRAINED-SOIL-CEMENT MIXTURES

Input area				Optimizing H-B fitting parameters			Optimizing target		
$\sigma=q_u$ (MPa)	c (MPa)	(deg)	Estimated GSI(Eq.11)	m_b	s	a	$(\tau_{MC}-\tau_{HB})=0$	$(\sigma_{n\_HB}-q_u)=0$	$\lvert\sigma-q_u\rvert+\lvert\tau_{MC}-\tau_{HB}\rvert=0$
1.5	0.386	43	40	13.132	0.014	0.638	2.11E-04	2.33E-06	2.13E-04
2	0.498	43		12.419	0.016	0.650	4.32E-03	3.26E-05	4.35E-03
2.5	0.611	43		12.267	0.013	0.652	1.15E-05	-1.10E-05	2.25E-05
1.5	0.386	43	45	13.744	0.001	0.628	-4.92E-08	-9.78E-03	9.78E-03
2	0.498	43		12.489	0.050	0.649	-1.83E-07	-3.23E-03	3.23E-03
2.5	0.611	43		12.223	0.020	0.653	3.41E-07	5.51E-05	5.54E-05
1.5	0.386	43	50	12.008	0.203	0.659	4.95E-07	-1.54E-04	1.55E-04
2	0.498	43		12.327	0.015	0.651	7.45E-03	2.36E-09	7.45E-03
2.5	0.611	43		11.574	0.132	0.666	4.82E-05	-3.84E-10	4.82E-05

TABLE III. PARAMETERS OF SOIL-CEMENT MIXTURES FOR H-B CRITERION

Strength Parameters			Fitting Parameters for H-B criterion				Other Parameters			
(deg)	Estimated GSI	q_u (MPa)	Initial m_b	Initial s	Residual m_b	Residual s	Elastic Modulus E/MPa	Poisson's Ratio	Unit Weight kN/m^3	Initial Earth Stress Ratio, K_0
43	40	1.5	13.132	0.014	13.132	0.014	1160.6	0.24	23	0.39
43		2.0	12.419	0.016	12.419	0.016	1340.2	0.24	23	0.37
43		2.5	12.267	0.013	12.267	0.013	1498.4	0.24	24	0.35
43	45	1.5	13.744	0.001	13.744	0.001	1160.6	0.24	23	0.39
43		2.0	12.489	0.050	12.489	0.050	1340.2	0.24	23	0.37
43		2.5	12.223	0.020	12.223	0.020	1498.4	0.24	24	0.35
43	50	1.5	12.008	0.203	12.008	0.203	1160.6	0.24	23	0.39
43		2.0	12.327	0.015	12.327	0.015	1340.2	0.24	23	0.37
43		2.5	11.574	0.132	11.574	0.132	1498.4	0.24	24	0.35

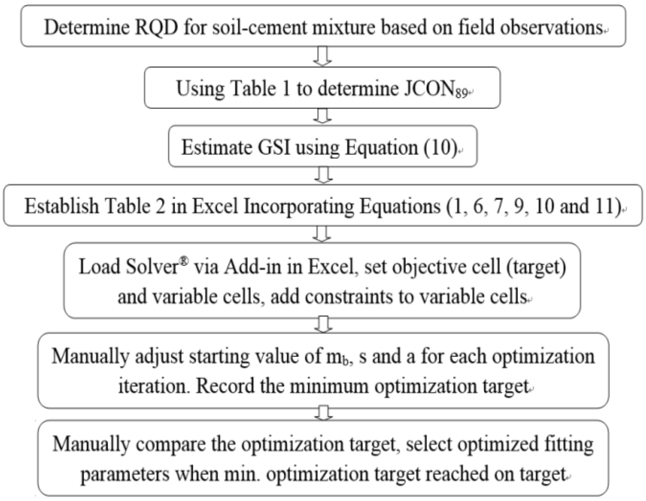

Figure 3. Flowchart of the procedure

V. NUMERICAL INVESTIGATION USING THE OPTIMIZED PARAMETERS

A. *Two-Dimensional Finite Element Model*

Wang et al. (2014) introduced a numerical study for an excavation project of a subway transfer station in Beijing, China. Fig. 4 shows the geological conditions of the project and Fig. 5 shows the cross-sections and method used for exacavations. They built a two-dimensional finite element model as shown in Fig. 6 with the commercial software Midas GTS® (version 4.0) developed by Midas Information Technology Co. Ltd, Korea (Midas 2013). To focus on applying the optimized parameters for the H-B criterion, the two-dimensional finite element model is re-used in this study but the H-B criterion is adopted only for grouting stabilized ground as shown in Fig. 7 where M-C criterion was used previously. All other issues such as simulation of excavation stages, boundary conditions, parameters of native soil for M-C criterion, etc. remain the same. For the stabilized area shown in Fig. 7, optimized fitting parameters of H-B criterion summarized in Table 2 are used. In addition, monitored data used in Wang et al. (2014) is also used in this investigation for comparisons.

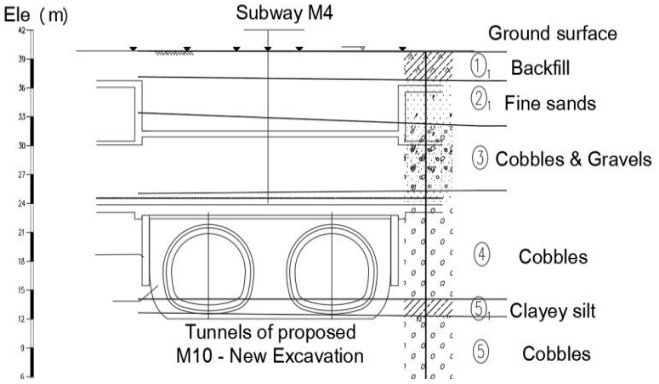

Figure 4. Geological conditions of the excavation case study (Wang et al. 2014)

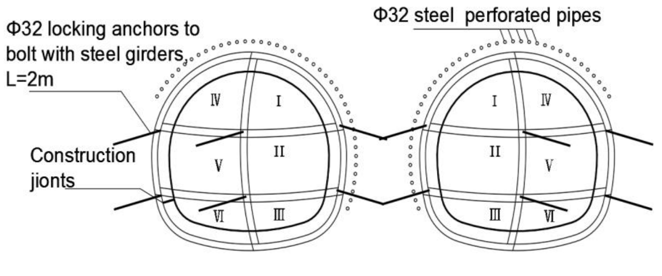

Figure 5. Cross-sections and excavation methods of the case study (Wang et al. 2014)

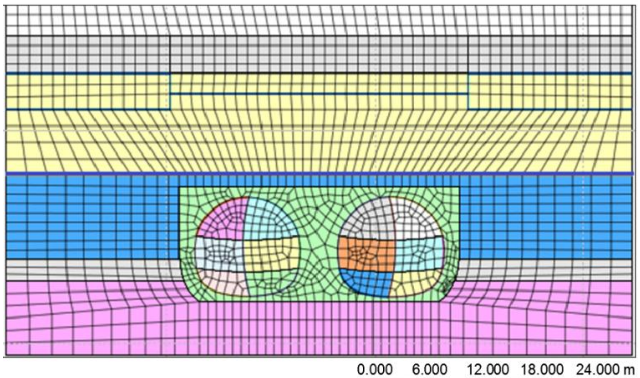

Figure 6. A two-dimensional FEA model used by Wang et al. 2014

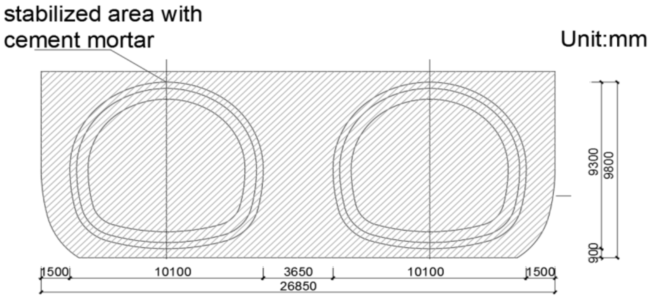

Figure 7. Area using grouting stabilized ground of the example

VI. RESULTS OF ANALYSES AND COMPARISONS

A. Evaluating H-B and M-C Criterion Applied to Soil-Cement Mixtures

Fig. 8, Fig. 9 and Fig. 10 show the finite element analysis results of ground surface subsidence using the H-B criterion with optimized parameters classified with estimated *GSI* =40, 45 and 50, respectively. From the view of the mechanism, the settlement is significantly related to deformation modulus which depends on *GSI* and σ_{ci} (i.e. q_u) as Equation (7) shows. Table 3 summarizes the parameter values for H-B criterion. Note that the elastic modulus of soil-cement in Table 3 is based on Wang et al. (2014) instead of using Equation (7) in this paper. The program requires input residual values for m_b and s. In order to compare the results using H-B and M-C criteria respectively, optimized values for m_b, s and a are used in these analyses to obtain Fig. 8, Fig. 9 and Fig. 10.

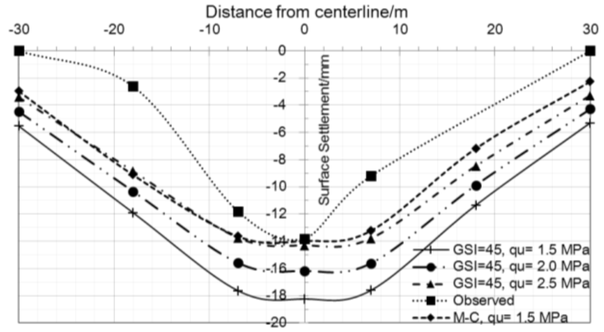

Figure 8. Surface settlements calculated with FEA using H-B (Estimated GSI =40) and M-C models

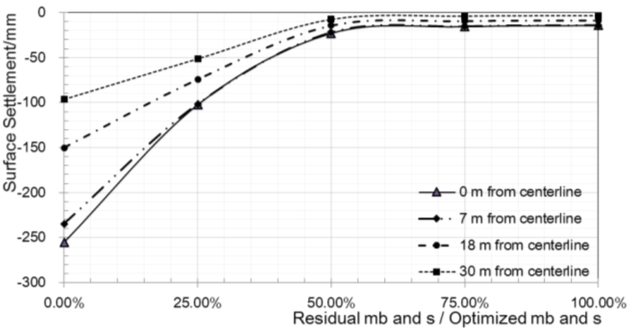

Figure 9. Surface settlements calculated with FEA using H-B (Estimated GSI =45) and M-C models

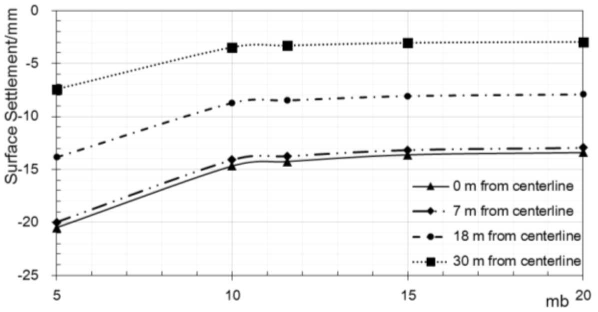

Figure 10. Surface settlements calculated with FEA using H-B (Estimated GSI =50) and M-C models

Fig. 8, Fig. 9 and Fig. 10 show the settlement curves match very well with field monitored data when using q_u=2.5 MPa, while previous simulation using M-C criterion indicated that q_u= 1.5 MPa matches well with the real monitored data (Wang et al. 2014). Note that although different GSI (40, 45 and 50) were used in the analyses, the influences on the calculated settlement from GSI are not significantly. This implies that when applying Hoek-Brown criterion to soil-cement mixture for settlement analysis, GSI may be ignored and a roughly reasonable number can be input instead.

Comparing the settlement curves using H-B criterion with the one using M-C criterion in Fig. 8, Fig. 9 and Fig. 10, it is found q_u=1.5 MPa applying to M-C criterion for soil-cement mixtures matches the observed settlement well, while the unconfined compressive strength of soil-cement mixture reaches 2.5 MPa to obtain the same goal when using H-B criterion. This implies that H-B criterion may overestimate settlement than M-C criterion when applying them to soil-cement mixtures.

B. Influences of Fitting Parameter m_b and s Values on Calculated Settlement

Fig. 11 shows the variation of calculated settlement with the used residual strength of fitting parameters m_b and *s*. Note that the excavation is approximately symmetric, calculation data on the right half to the centerline were read. When performing analyses, the residual values for m_b and *s* increase from the program default values (approximately 0% of optimized m_b and *s*) to 25%, 50%, 75% and 100%, while other parameters remain unchanged (using parameters for q_u=2.5 MPa and GSI=45). We can find the calculated settlement goes to unrealistic large when very small residual values of m_b and *s* were used. With the increase of residual m_b and *s*, the calculated settlement generally decreases. When the residual m_b and *s* reaches 50% of the optimized values, the settlement values become flat, which means they won't significantly affect the magnitude of calculated settlement anymore.

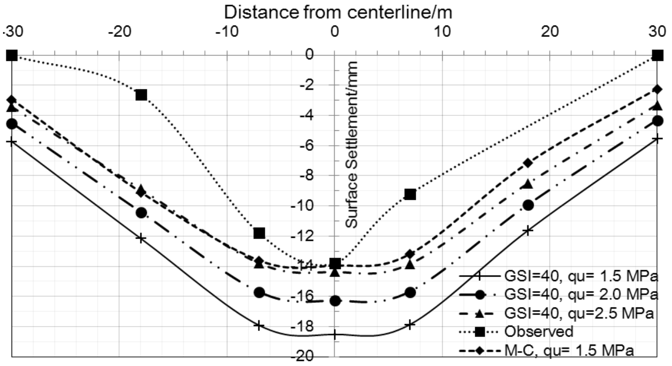

Figure 11. Influence of residual m_b and *s* on calculated surface settlement using H-B criterion

Hoek and Marinos (2000) indicated that constant m_i that defined the friction characteristics of the component materials in these rock elements is very important (obviously E_m is important for deformation). As Equation (4) shows, the fitting parameter of m_b for the H-B criterion is related to *GSI* and m_i, however, m_i was not used in the proposed optimization procedure which is one of the advantages of the proposed method to save the efforts to run triaxial tests to obtain it, and GSI is not significant when applying H-B to soil-cement mixture. To explore the influence of single m_b on the calculated settlement using H-B criterion, numerical analyses were carried out with a series of m_b values 5, 10, 15 and 20, whereas other optimized values remain the same for q_u=2.5 MPa and GSI =45. Fig. 12 shows the variation of the calculated ground surface settlement in accordance with the values of m_b at different monitoring points. It is found that the calculated surface settlement decreases with an increase of m_b value. Moreover, the surface settlement is more sensitive to the m_b when m_b< 10. The curves also show a trend that the influence of m_b is negligible when m_b>15.

Similar analyses were performed to explore the influences of single *s* values on the calculated settlement using H-B criterion. The results are shown on Fig. 12. We can find that the calculated settlement generally decreases with an increase of *s* values, but it seems that *s* has more influences on the calculated settlement with an increase of offset from the centerline.

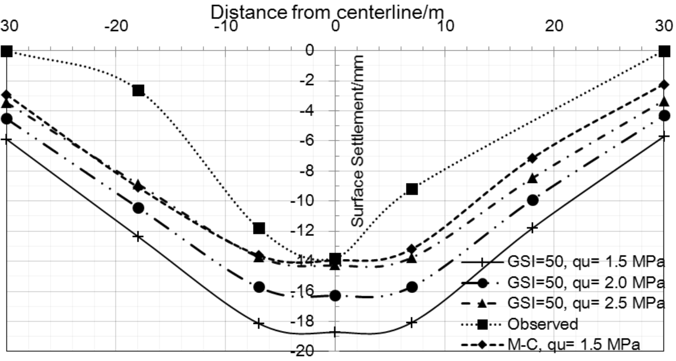

Figure 12. Influence of m_b values on the calculated settlement

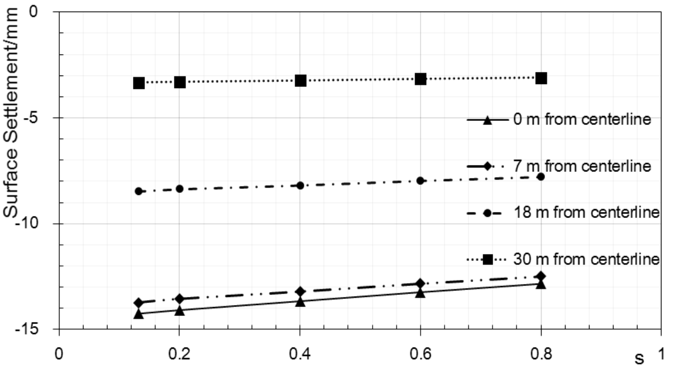

Figure 13. Influences of *s* values on the calculated settlement

VII. CONCLUSIONS

This study presents an optimization-based approach to estimate the fitting parameters of the H-B criterion from M-C shear strength parameters for soil-cement mixtures. Based on the graphic expressions of the two criteria and analytical solutions, Solver® in Excel® is used to process the optimization. As an example and an approximate validation, numerical investigation using the H-B criterion with optimized parameters for coarse-grained-soil-cement mixtures is conducted. Conclusions of this study are summarized as follows:

- The proposed procedure can be used to estimate the parameters of the H-B criterion for soil-cement mixture without a strong knowledge of geology. *mi* for intact rock is not needed in optimization procedure, GSI is not so significant for soil-cement mixtures as rocks when applying H-B criterion in numerical analysis. All parameters needed are friction angle ϕ, the cohesion c and the unconfined compressive strength *qu* of the soil-cement material. Approximate GSI can be estimated with the procedure.
- The optimized fitting parameters of H-B criterion, m_b, s and a are the starting values for finite element input using H-B criterion. Very small m_b, s and a will lead to unrealistic large settlement, and the calculated settlement decreases with an increase of residual values of these fitting parameters. When reaches 50% of optimized fitting parameters, the influences will become less significant, and the calculated settlement curves become flat. Thus residual values between 50% and 75% of the optimized fitting parameters of H-B criterion for soil-cement mixtures are recommended for finite element analysis.
- Calculated settlement decreases with increase of single m_b value for soil-cement mixture. The calculated surface settlement is more sensitive to the m_b when $m_b< 10$, the influence of m_b is less significant when $m_b>15$. The calculated settlement also decreases with an increase of s values, but s has more influences on the calculated settlement with an increase of offset from the centerline.

Compared with previous numerical analysis using M-C failure criterion for coarse-grained-soil-cement mixtures, it seems that numerical analysis results using the H-B criterion are more conservative.

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REFERENCES

[1] American Society of Civil Engineers (1995). Verification of geotechnical grouting, Committee on Grouting. Geotechnical Special Publication No. 57, ASCE Press, Reston, VA, 77p.

[2] Bae, G.J., Shin, H.S., Sicilia, C., Choi, Y.G. & Lim, J.J. (2005). Homogenization framework for three-dimensional elastoplastic finite element analysis of a grouted pipe-roofing reinforcement method for tunneling. Int. J. Numer. Anal. Met. , 29(1), 1-24.

[3] Balmer, G.G. (1958). Shear strength and elastic properties of soil-cement mixtures under triaxial loading. Reprint from the copyrighted Proc. of American Society for Testing Materials by Portland Cement Association, Research and Development Laboratories, pp.1187-1204.

[4] Barton, N. (2013). Shear strength criteria for rock, rock joints, rock fill and rock masses: Problems and some solutions. J. Rock Mech. Geotech. Eng., 5,249-261

[5] Barton, N. & Bandis, S. (1982). Effects of block size on the shear behavior of jointed rock. Proc. of the 23rd US Symposium on Rock Mechanics, Berkeley, California, pp. 739-760.

[6] Bieniawski, Z.T. (1989). Engineering rock mass classifications, Wiley, New York, 251p.

[7] Eberhardt, E. (2012) The Hoek–Brown failure criterion. Rock Mech Rock Eng., 45,981-988.

[8] Hoek, E. & Brown, E.T. (1988). The Hoek–Brown failure criterion-a 1988 update. In: Curran J (ed) Proc. 15th Canadian Rock Mechanics Symposium. University of Toronto, Toronto, pp. 31-38.

[9] Hoek, E., Carranza-Torres, C.T. & Corkum, B. (2002). Hoek–Brown failure criterion-2002 edition. Proc. 5th North American Rock Mechanics Symposium (NARMS-TAC), University of Toronto Press, Toronto, pp. 267-273.

[10] Hoek, E., Carter, T.G. & Diederichs, M.S. (2013). Quantification of the geological strength index chart. 47th US Rock Mechanics/Geomechanics Symposium, San Francisco, CA, USA.

[11] Hoek, E. &Marinos, P. (2000). Predicting tunnel squeezing problems in weak heterogeneous rock masses. http://www.rockscience.com/hoek/refernces/H2000.pdf, March 7, 2004.

[12] Kumar, P. (1998). Shear failure envelope of Hoek–Brown criterion for rock mass. Tunn. Undergr. Space Tech., 13(4), 453-458.

[13] Labuz, J.F. & Zang, A. (2012). Mohr–Coulomb failure criterion. Rock Mech. Rock Eng., 45, 975-979.

[14] Midas (2013). User's Manual for Geotechnical and Tunnel Structures. Midas Information Technology Co. Ltd, Korea.

[15] Mitchell, J. K. (1976). The properties of cement-stabilized soils. Proc. Residential Workshop on Materials and Methods For Low Cost Road, Rail, and Reclamation Works, Leura, Australia, pp. 365-404.

[16] Mollon G., Dias, D. & Soubra, A.H. (2013). Probabilistic analyses of tunneling-induced ground movements. Acta Geotechnica 8(2), 181–199

[17] Shen, J.Y., Priest, S.D. & Karakus, M. (2012). Determination of Mohr–Coulomb shear strength parameters from generalized Hoek–Brown criterion for slope stability Analysis. Rock Mech. Rock Eng., 45, 123-129.

[18] Terzaghi, K. (1946). Rock defects and loads on tunnel supports, Harvard University, Cambridge, MA, USA, 95p.

[19] Wang, D., Xing, X., Qu, H. & Zhang, L.M., (2015). Simulated radial expansion and heave caused by compaction grouting in non-cohesive soils. Int. J. Geomech,. doi:10.1061/(ASCE)GM.1943-5622.0000333.

[20] Wang, D., Olowokere, D.O. & Zhang, L.L. (2014). Interpretation of soil-cement properties and application in numerical studies of ground settlement due to tunneling under existing metro. Geotech. Geolog. Eng., doi: 10.1007/s10706-014-9803-2.

[21] Yoo, C. (2002). Finite element analysis of tunnel face reinforced by longitudinal pipes. Comput. Geotech., 29,73-94.

[22] Yoo, C. &Shin, H.K. (2003). Deformation behavior of tunnel face reinforced with longitudinal pipes-laboratory and numerical investigation. Tunn. Undergr. Sp. Tech., 18, 303-319.