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Development of Mathematical Model for Optimal Planning Area Allocation of Multi Crop Farm

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Abstract- For irrigation projects with limited water resources, or restricted supplies from pump capacities irrigation management decisions need to be taken well before the irrigation season. Irrigation managers need to anticipate crop selections, plan for crop rotations, and project water deliveries to each crop. To take such decisions a water allocation model has been built to determine optimal crop area to cultivate by each crop among crop mix of two to six crops under the constraint of prevailing climate conditions, available water resource and critical command area with the objective of attaining the optimum critical area.

The mathematical model of the problem is linear in nature subject to various constraints due to availability of total land area, water. Consequently, linear Programming was used in this study to make decisions about irrigation water management options in conjunction with optimal cropping patterns to ensure optimal use of water. The developed model is also capable of matching scheme water demand and supply through many different scenarios including: staggering of the sowing date, increasing scheme water supply, changing the operational criteria of the scheme, changing crop variety and eventually increasing the overall irrigation efficiency. The computer model's accuracy has been validated by taking different water management scenarios under the prevailing conditions of Rahad Irrigation project.

Keywords- *Multi-crop plan, Decision model, Decision-aid, and Water Use Efficiency*

I. INTRODUCTION

Crop production is a complex enterprise involving many decision-making processes that depend on a host of factors. Some factors, like climatic conditions, land characteristics, etc., are inherent to the farm and cannot be altered or controlled. Other farm properties, like the current structure of the machinery stock and personnel, the irrigation infrastructure in place, etc., are factors possibly to be taken into account. According to Recio, et al. (2003) these factors can be modified for the purposes of achieving maximum profitability. The above factors are what constitute the farm's options. These options cover a wide variety of alternatives on which decisions have to be made, such as the choice of which crops to grow, which field operations to perform, how and when to complete these operations, using which machinery, which fertilizers and other chemical substances are to be applied, etc. This is what is known as field operation planning. Therefore, the field operation planning problem is inseparable from any analysis involving activity scheduling and cost control.

As reported by Allen (1998), a cropping pattern indicates the kind and sequence of crops grown over a period of time on a given area of land. Cropping patterns are determined by agro-climatic and socio-economic factors. Generally, agro-climatic factors are fairly stable over time, while demographic, social and economic factors are less stable. Agro-climate factors determine the condition under which crops are grown. On the other hand, farmers are increasingly inclined to change cropping patterns in response to changes in economic factors (input-output prices), technological factors (improved efficiency), institutional factors, and policy related factors (prices, irrigation subsides or charges). Zhenmin, (1994)

Sudan being an example of the sub-Sahara African countries, irrigation schemes is envisaged to be water limited, mainly due to poor performance of water resource utilization which is attributed to the system deterioration, change in project goals and the economic and social pressures. McDonald, (1992)

Certainly, the critical issue in improving performance of canal system is the relative priority assigned to structural and non-structural measures. Structural improvements (such as canal lining, new flow control structures, land leveling) are generally the most popular options particularly to government agencies; however, these improvements are considered as the most expensive ones. Nevertheless, increased emphasis on operation and maintenance (operation plan) and turnover management system (water users association or privatization) are often more efficient than modification to the infrastructure. Even in areas with adequate water supply such as the case of Rahad Irrigation Project in Sudan, any success depends on a high level of management of the distribution system.

This study attempted to examine how irrigation water can be optimally allocated to crops under different policies, and how these polices influence the size of area to irrigate without exceeding the available water supply. The objective of this study is to develop a strategic simulation and optimization model for evaluation of optimal cropping plan according to farm water and area constraints.

II. MATERIALS AND METHODS

In this study a linear programming based optimization model is used for crop planning. The linear programming model was essentially static, allocating irrigation water in a single year among different crops. The model maximizes net returns from crops and yields optimal crop plan and monthly releases required from reservoir. The model constraints are Surface water, land availability. The problem statement can be captured in three main questions: how should water be allocated to crops? What is the actual irrigation water demand? What amount of land and water resources should be devoted to each crop?

For the purpose of modeling, Turbo Pascal language (ver.7) was chosen as a programming language due to its adaptability, easiness and structural nature that minimizes consumption of memory capacity. The model falls within "menu driven" program where a menu based menu-interface is used to control the whole sequence of program's operations. The program (HEWASP) main body consists of a master program (Pen.pas), four major subsidiary parts (Units) (Penman.Pas, Manpunit.Pas, Planmode.Pas and Smplx.Pas) and two built-in data files (Stages.Dat and Factors.Dat). Most of these parts are dedicated to carry specific functions. The program consists mainly of Critical Command Area Estimation Module and Crop Planning Module.

III. THE MATHEMATICAL FORMULATION OF THE MODEL

For the purpose of this study, the objective function of the linear programming model is to maximize the total cultivated area of the project, the formula for which is as follows:

$$Max\ Z = (1/2)\ \sum_{i=1}^{N} Ai/Ci \qquad (1)$$

Where:

Z: total planned area to be cultivated during the critical decade (ha),

Ai: optimum area of crop (i), (output of the model (ha)),

N: number of crops,

Ci: coefficient relating the maximum planned area of crop (i) to the total planned area to be cultivated during the critical decade, thus:

$$Ci = (PA\ i\ max / Z) \qquad (2)$$

Where:

PA i max: maximum planned area of crop (i) (ha).

In calculation of the optimum crop area beside the maximum objective function, the relevant constraints must be included. They are as follows:

i- Constraints of Total Area: The combined crop area irrigated for all crops should be equal to or smaller than the total project area that can be irrigated safely (project critical area). This in fact expresses the allocation of limited water through the critical area between the crops grown in the project so:

$$\sum_{i=1}^{N} Ai \leq CRA \qquad (3)$$

Where:

CRA: critical area of the project during critical decade (ha).

ii- Constraints of Bounds on Maximum and Minimum Area under Various Crops: The state's strategy for food security and its export-import policies dictate the maximum (PA i max) and minimum (PA i min) area to be allotted for each crop. Hence, this constraint can be stated mathematically as follows:

$$PAi\ \mathrm{min} \leq Ai \leq PAi\ max \qquad (4)$$

iii- Constraints of Water Supply: The available water supply during the critical decade limits the size of project area to be cultivated. The crops demand must match the available supply in the critical decade as expressed by the following relation:

$$SWSc \leq \sum_{i=1}^{N} INgi \cdot Ai \qquad (5)$$

Where:

SWSc: scheme water supply during the critical decade (m^3/decade),

INg i: gross irrigation water need for crop (i) during critical decade (m^3ha/decade).

The mathematical formulation of the objective function and its constraints is made as follows:

$$Min\ CWR = \sum_{i=1}^{nd} CWRai + \sum_{i=1}^{nd} CWRbi + \sum_{i=1}^{nd} CWRci \qquad (6)$$

Subject to:

$$\begin{array}{l} \mathrm{X1CWRa\ 3 + X2CWRb\ 2 + X3CWRc\ 1 \geq CWRb\ 2} \\ X1CWRa\ 4 + X2CWRb\ 3 + X3CWRc\ 2 \geq CWRb\ 3 \\ \quad . \qquad . \qquad . \qquad . \\ \quad . \qquad . \qquad . \qquad . \qquad (7) \\ \quad . \qquad . \qquad . \qquad . \\ X1CWRa(nd) + X2CWRb(nd-1) + X3CWRc(nd-2) \geq CWRb(nd-1) \end{array}$$

Where:

CWRa: water requirement of the crop planted in the decade before the optimum planting decade (m^3ha/decade),

CWRb: water requirement of the crop planted during the optimum planting decade (m^3ha/decade),

CWRc: water requirement of the crop planted in the decade after the optimum planting decade (m^3ha/decade),

X_1, X_2 and X_3: percentages of planned area is planted during a decade before, within and after optimum planting decade respectively, nd: number of decades during the growing season.

The critical command area estimation module is composed of:

i- Calculating reference Evapotranspiration per decade (ETo) using Penman-Monteith Method (1998) as:

$$ETc = ETo \cdot Kc \quad (8)$$

Where:

ETo = evapotranspiration,

Kc = crop coefficient.

ii- Determination of Net Irrigation Water Need (INnet) by comparing (ETc) to effective rainfall (ER) as:

$$INnet = ER \stackrel{\text{def}}{\equiv} ETc \quad (9)$$

iii- Determination of Gross Irrigation Water Need (INg) by considering impacts of project efficiency (PE) on Net irrigation water need (INg) as:

$$INg = INnet/PE \quad (10)$$

iv- Determination of Scheme Water Demand (SINg) per decade for cases of multi-cropping, is calculated according to Doorenbos and Pruitt, (1977) as an sum of weighted averages of each crop based on its relative area (Ra) and Gross irrigation water need (INg) as:

$$SINg = \sum INg \cdot Ra \quad (11)$$

v- Estimation of operational scheme irrigation needs (SINop) (m^3/ha/decade), from determination of Scheme Water Demand (SINg) per decade and time of operation of the water capture plant at the source (Top) as suggested by Doorenbos and Pruitt (1977):

$$SINop = SINg/Top \quad (12)$$

vi- Estimation of Scheme Water Supply (SWS) per decade depends primarily on the availability of the water at its source, capacity of the conveyance system from the water source, and the reliability of scheme water supply.

vii- Determination of Command Area (CA in ha)) and Critical Area (CRA): When the supply of a water source in a certain decade(SWS in m3 decade-1) and the average gross irrigation need per hectare are known (INgavg in m3 decade-1ha-1), then the command area can be calculated as:

CA=SWS / INg. (13)

The Selection of Decision Variables demented on the construct of the linear programming optimization model, the algebraic variables will be assigned to the various parameters of water allocated as the main field crops of Project the flow chart of the program is depicted in fig. 1.

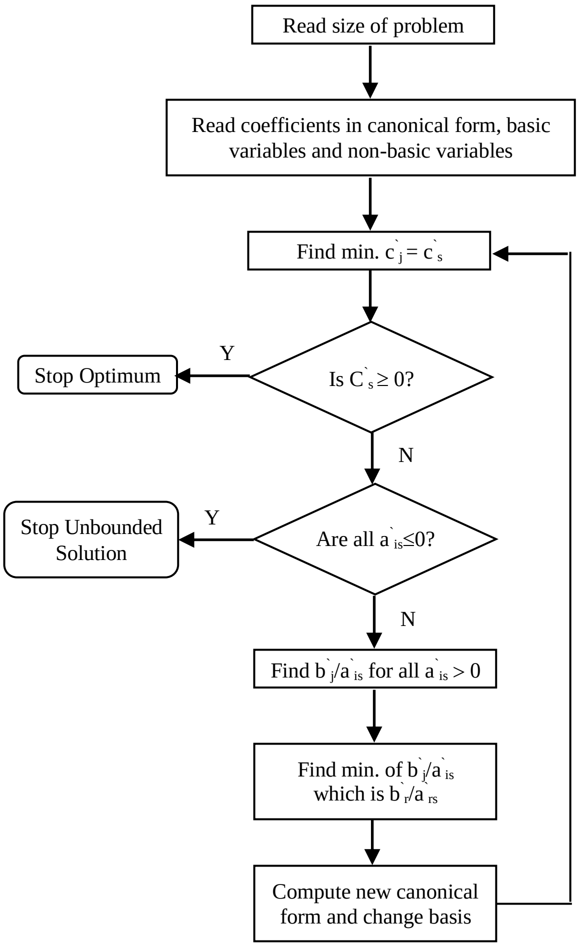

Figure 1. Flowchart of the program (HEWASP)

IV. Field Data Setting

For the purpose of validation model the required data is obtained from both primary and secondary sources from Rahad Irrigation Scheme. The project lies at a distance of 160 km south east Khartoum in the central clay plain of the Sudan. It extends between latitudes 13° 43` N and 14° 35` N and between longitudes 35° 55` E and 34° 22` E at an elevation varying between 400.00 - 430.00 m above mean sea level. It is about 140 km in length and 25 km in width. It lies along the eastern bank of the River Rahad, which is a tributary of the Blue Nile, originating from the Ethiopian plateau. The project area has semi-arid tropical climate with a humid rainy season extending from June to September followed by a dry period from October to May. The annual rainfall varies from 350 mm in the north to 650 mm in the south (Elramlawi, 1992).

V. RESULTSANDDISCUSSIONS

A. *Model Evaluation*

According to Dent et al. (1979), model evaluation is a two-fold process. It involves model verification and validation.

B. *Model Validations*

In order to test the model adequacy, subjective assessment of the model will be made in relation to the model purposes and not from absolute point of view (Dent et al., 1979). As given in the study specific objectives, the program main purpose is to be used as a planning tool through estimation of critical area, allocating of this critical area between crops and matching scheme water supply and demand. Validity of matching supply and demand in terms of changing crop rotation, improving irrigation efficiencies and increasing scheme water supply under stochastic rainfall will be made on subjective basis. However, to aid this, two qualitative indicators and regression analysis will be employed. These indicators are:

1) *Modified Water Delivery Performance (MWDP)*

Bailey and Lenton (1984) suggested the water delivery performance (WDP) to quantify the relationship between actual water supply and actual water needed. WDP, in its original form, reflects only cases of optimum and non-optimum water supply. In this study, and in order to reflect different cases of water supply that may occur under field conditions throughout the growing season (over, optimum or under-irrigation of water supply), it has been modified (fig. 2) to be as follows:

$$MWDP = Max\ (SWRt / SWSt) \quad t = 1 \ldots nd \qquad (14)$$

Where:

SWRt: scheme water required (or needed) during decade (t) (m^3/decade).

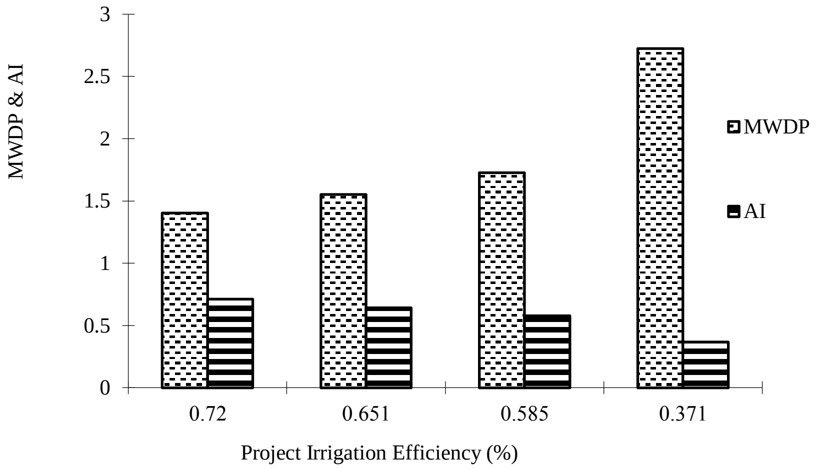

Figure 2. Effect of changing project irrigation efficiency on modified water delivery performance (MWDP) and area indicator (AI) under project assumed design conditions

2) *Area Indicator (AI)*

$$AI = (CRA / PA) \qquad (15)$$

Where:

CRA: is the critical area of the project during critical decade (ha),

PA: is the planned area to be cultivated during the critical decade (ha).

Note that not all the project area is the planned area during the critical decade but planned area refers to the area of those crops planned to be grown on the stated decade.

Recall that (AI) is a function of plant demand as expressed by (INg) and the existing water supply (SWS). Hence, the area indicator was intended to be used for evaluating the planning scenario under the following cases:

If AI = 1.0: then plan for water demand matches well with the existing water supply and indicating a situation of no problem.

If AI < 1.0: then a problem arises and the plan of water needs is to be revised in order to match the existing water supply. This is because some of the intended area cannot be irrigated to its required satisfaction.

If AI > 1.0: then more area can be cultivated due to availability of excess unutilized water. However, in this case, at least all farmers have equal chance to cultivate this area. For estimation of critical area, allocating of this critical area between crops and matching scheme water supply and demand, the description of different scenarios and their groups is given in table 1.This is made to determine effects of: Crop Rotation, efficiency, SWS, and rainfall on design and Existing Conditions.

TABLE I. DESCRIPTION OF DIFFERENT SCENARIOS AND THEIR GROUPS

Scenario No.	Course Rotation	Efficiency (%)	Effective Rainfall (mm decade^{-1})	SWS (Mm^3 decade^{-1})	Scenario No.	Course Rotation	Efficiency (%)	Effective Rainfall (mm decade^{-1})	SWS (Mm^3 decade^{-1})
1	4	65	Normal	8.4	8	4	59	Normal	Existing
2	2	65	Normal	Existing	9	4	59	Normal +30%	Existing
3	3	65	Normal	8.4	10	4	59	Normal -30%	Existing
4	2	65	Normal	8.4	11	4	72	Normal	Existing
5	4	37	Normal	Existing	12	4	72	Normal +30%	Existing
6	4	37	Normal +30%	Existing	13	4	72	Normal -30%	Existing
7	4	37	Normal -30%	Existing	14	4	65	Normal	Existing

As given in fig. 3 the area indicator (AI) shows that in the 2 course and 3 course rotations about 9% of the area planned cannot be cultivated. The introduction of 3 course rotation did not improve the area lost in 2 course rotation. In contrast to that, with the 4 course rotation more than 30% of the planned area can be cultivated. It is to note that if the Rahad authority had realized this and recognized the causes of water shortage and had the tool to assess the discrepancy between demand and supply made with each rotation, they would have not used the 3 course rotation for eight consecutive years at all.

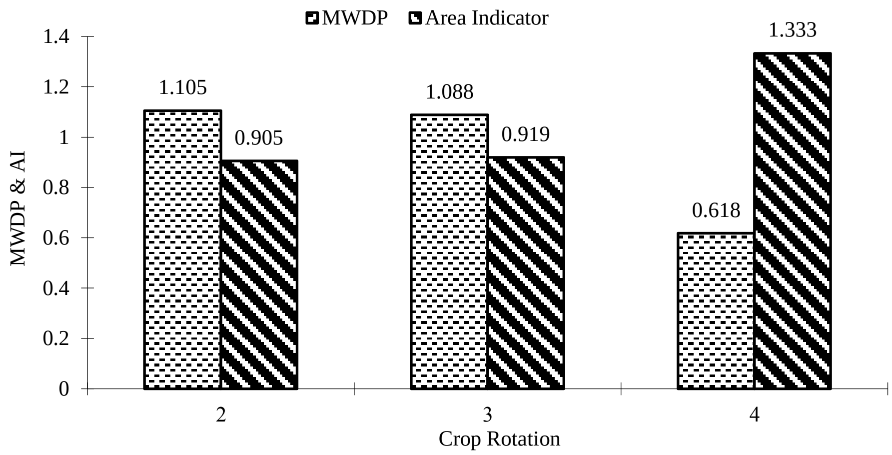

Figure 3. Effect of changing crop rotation on modified water delivery performance (MWDP) and area indicator (AI) under project assumed design conditions

It can also be shown from Figure (3) that MWDP decreases with increasing the number of crops in the rotation. The decrease in water demand can be justified by the reduction of the relative area allocated for crops with high water requirement. In addition, introduction of winter crop in the four-course rotation sharply reduced the demand for water. The reduction was found to be due to reduction of number of crops cultivated during the critical decade. The feasibility study made by Macdonald and Partners (1976) considered 32 m^3 fed^{-1} day^{-1} as a peak demand for establishing Rahad Project. On the other hand, Farbrother (1977) reported a peak demand of 35 m^3 fed^{-1} day^{-1} for cotton crop during the third decade of September in Gezira Project. Rahim (1999) reported that the capacity and number of pumps in the Rahad Project were determined according to a peak cotton crop requirement (28 m^3 fed^{-1} day^{-1}). Therefore, the designed scheme water supply would amount to 8.4 mm^3 day^{-1} (300,000 fed at 28 m3 fed^{-1} day^{-1}). However, a peak cotton crop water demand of 31 m^3 fed^{-1} day^{-1} was obtained by using the FAO-CROPWAT in (1991). When the (HEWASP) was employed, a value of 30 m^3 fed^{-1} day^{-1} was obtained, which is in agreement with Adam (1996). One avenue to recover the water deficit and match the supply and demand given in this study was to change the crop type or its area. This is possible by changing the crop rotation.

VI. CONCLUSIONS

In the present study validation of HEWASP model is shown for determining the optimal cropping pattern so as to maximize total project area to cultivate. Hence, the overall view of this study can be summarized as follows:

1- The adopted modified water delivery performance index (MWDP) reflects the cases of optimum, under and over-irrigation, while water delivery performance index (WDP) reflects only the optimum and non-optimum cases.

2- To minimize watering problems, it is suggested to revise the design criteria of Rahad Irrigation Project canalization network depending on the concept of reference evapotranspiration as reviewed by FAO, (1998).

3- HEWASP can be employed as a pre-seasonal planning tool to predict the area to be cultivated at each level of project overall irrigation efficiency. The project overall irrigation efficiency can be related to the management cost to aid making the decision on the most feasible area to be cultivated especially at the time of budget estimation.

4- An operating procedure to manage irrigation water in order to maintain the level of the area planned to be cultivated is suggested.

5- Staggering of the planting period (sowing date) can be used as an alternative, among others, to improve the area planned to be cultivated.

6- The linear programming technique (LR) is useful in allocating critical area and staggering process.

7- Further studies are recommended to be conducted to investigate the effect of different staggering periods on crop yield.

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