

# Coefficient of Isothermal Gas Compressibility for Reservoir Fluid in Niger Delta Region

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**Abstract-** The Isothermal gas compressibility is a point function and has the dimensions of reciprocal pressure. This property is usually determined from various equations of state developed from other western country's field data. This study is based on developing a new correlation and using the correlation to obtain a data base for the isothermal gas compressibility coefficient based on the Niger Delta hydrocarbon fields.

The Ghedan et al formular for finding the dimensionless Isothermal gas compressibility coefficient in combination with the Beggs and Brill z factor correlation was used in obtaining the various values of the Isothermal gas compressibility coefficient. The values obtained were used to develop a chart of dimensionless isothermal gas compressibility versus pseudo-reduced pressure at a constant pseudo-reduced temperature.

Evaluation of the obtained result was carried out using statistical methods and the ranking method. It was observed that this study had a better statistical evaluation with average absolute percentage error of 5.34 and correlation coefficient of 0.94 as compared to other correlations. Also, this study had the lowest rank of 2.42 with Mattar et al correlation having the highest rank of 4.94. Therefore, this agrees with literature that, the lower the rank value, the better the correlation.

The better evaluation of this study can be linked to the more accurate EOS model of the z factor applied which requires no iteration, consuming less time in arriving at the desired result and this procedure can be easily programmed.

## I. INTRODUCTION

Natural gas is a mixture of hydrocarbon compounds and because this mixture is varied in types as well as relative amounts of the compound, the overall physical property will vary.

Methods for predicting reservoir performance, particularly those based on the compositional material balance, depends upon the capability of accurately expressing the molar volumes and other fluid properties as a function of temperature, pressure and composition, (Surjit, 1968). Unlike oil, natural gas properties vary significantly with pressure, temperature and gas composition. The isothermal compressibility coefficient is defined as the volumetric change when pressure is changed and temperature is held constant. For an ideal gas, it is defined as

the inverse of pressure making it a point function that has the dimension of reciprocal of pressure. For a real gas situation, the z factor is taken into consideration. Various authors like Trube, Matter et al e.t.c sort for simpler mathematical correlations in expressing the isothermal compressibility coefficient. These findings are based on Standing and Katz z factor chart or other developed modified correlations from Standing and Katz z factor chart.

Using the concept of pseudo-reduced compressibility, Trube obtained a correlation between isothermal gas compressibility coefficient and Standing and Katz chart (Vidar, 2006). Other authors like Mattar et al used the concept of pseudo-reduced compressibility and obtained a correlation between isothermal gas compressibility and the Dranchuk et al z factor correlation. Ghedan et al also developed a correlation by employing Dranchuk et al z factor correlation (Ghedan et al, 1991).

The coefficient of isothermal compressibility is an essential controlling factor in identifying the type of reservoir fluid and it's also important in some engineering calculation such as well testing and metering, analysis of production performance of gas wells etc. Several graphical and empirical correlations have been developed to determine this coefficient based on other western country's field data. Almost all Nigerian gases are gotten from the Niger Delta region. Therefore, the need to have an easy correlation for obtaining this measured physical property is required.

The purpose of this paper is centered on the development of a new and easy procedure of finding the isothermal gas compressibility coefficient by employing the concept of dimensionless isothermal gas compressibility coefficient developed by Ghedan et al and Beggs and Brill z factor correlation. This is done by collating PVT data reports from the Niger Delta region and validating all the obtained reports. Use the valid reports to develop a new data base in obtaining the isothermal gas compressibility coefficient within the Niger Delta region and evaluate the result obtained by using some statistical methods and the ranking method.

## II. METHODOLOGY

### A. Data for Study

PVT analysis report from 198 wells in the Niger Delta region was collected. These reports were analyzed for validity. The flash compositions of the gas, liquid and recombined in the reports, were used for the validation of the reports. From the 198 PVT reports collected, 72 was found to be valid and these valid reports were collected and the depletion study contained in them was used for the isothermal gas compressibility coefficient study.

### B. Validation

This is done to check the accuracy of reports and two methods were used. The material-balance diagram and the Campbell diagram (Buckley plot) (Ikiensikimama, 2009).

#### 1) The Campbell diagram (Buckley plot)

This technique describes the linear relationship between the log of the individual mole fraction of a component in the gas phase divided by the mole fraction of the same component in the liquid phase (k-values) and their respective temperature squared.

#### 2) Material Balance diagram

This diagram is based on the following general equation describing the individual component material balance around a flash separator stage:

$$F_i z_i = L_i X_i + V_i Y_i \quad (1)$$

$$\frac{Y_i}{z_i} = -\frac{L X_i}{V z_i} + \frac{1}{V} \quad (2)$$

A plot of  $Y_i/z_i$  versus  $X_i/z_i$  should result in a straight line of gradient  $-L/V$ .

Where F, L and V are the molar flow rates of feed, flashed liquid and flashed vapour respectively.  $z_i$ ,  $X_i$  and  $Y_i$  are the mole fraction of component  $i$  in the feed, flashed liquid and flashed vapour respectively. A deviation in the mass balance diagram from linearity indicates an arithmetic error in the data recombination.

### C. Isothermal Gas Compressibility Coefficient Study

The composition of the gas mixtures of the various valid reports at different depletion pressures were collected and placed in an excel format to generate the pseudo-reduced pressures and temperatures.

The pseudo-critical pressures and temperature was calculated using the equation below

$$P_{pc} = \sum_{i=1}^n P_{ci} Y_i \quad \text{and} \quad T_{pc} = \sum_{i=1}^n T_{ci} Y_i \quad (3)$$

and also corrected due to the presence of the non-hydrocarbon gases using the Carr et al method as given below.

$$P'_{pc} = T_{pc} - 80y_{CO_2} + 130y_{H_2S} - 250y_{N_2} \quad (4)$$

$$T'_{pc} = P_{pc} + 440y_{CO_2} + 600y_{H_2S} - 170y_{N_2} \quad (5)$$

This method was selected because it takes into consideration the presence of the nitrogen gas in the mixture.

The dimensionless isothermal compressibility coefficient of a real gas defined by Ghedan et al was used to obtain the values of the dimensionless isothermal gas compressibility coefficient.

$$c_g = \frac{c_r}{P} \quad (6)$$

Where

$$c_r = c_g P = 1 - \frac{P_{pr}}{z} \left( \frac{\partial z}{\partial P_{pr}} \right)_{T_{pr}} \quad (7)$$

$c_r$  = Dimensionless Isothermal gas compressibility coefficient

$P$  = Pressure, psi

$P_{pr}$  = Pseudo-reduced pressure,

$z$  = Compressibility factor, dimensionless

and the Beggs and Brill  $z$  factor correlation

$$Z = A + (1 - A)e^{-B} + CP_{pr}^D \quad (8)$$

Where:

$$A = 1.39(T_{pr} - 0.92)^{0.5} - 0.36T_{pr} - 0.101$$

$$B = (0.62 - 0.23T_{pr})P_{pr} + \left[ \left( \frac{0.066}{T_{pr} - 0.86} \right) - 0.037 \right] P_{pr}^2 + \left[ \frac{0.32}{10^9 (T_{pr} - 1)} \right] P_{pr}^6$$

$$C = (0.132 - 0.32 \log T_{pr})$$

$$D = 10^{0.3106 - 0.49T_{pr} + 0.1824T_{pr}^2}$$

The Beggs and Brill (1978)  $z$  factor correlation was differentiated to obtain the differential factor in the dimensionless isothermal compressibility equation above and it is given as

$$\frac{\partial z}{\partial P_r} = -[E + 2FP_r + 6GP_r^5][1 - A]e^{[EP_r + FP_r^2 + GP_r^6]} + DCP_r^{D-1} \quad (9)$$

Where

$$E = 0.62 - 0.23T_{pr}$$

$$F = \left( \frac{0.066}{T_{pr} - 0.86} \right) - 0.037$$

$$G = \frac{0.32}{10^9 (T_{pr} - 1)}$$

$$A = 1.39[T_{pr} - 0.92]^{0.5} - 0.36T_{pr} - 0.101$$

$$C = 0.132 - 0.32 \log T_{pr}$$

$$D = 10^{0.3106 - 0.49T_{pr} + 0.1824T_{pr}^2}$$

The above formulas were used in the entire valid wells at the various pseudo reduced pressures at constant pseudo reduced temperature to obtain the value of the dimensionless isothermal gas compressibility coefficient, cr.

#### D. Evaluation

To evaluate the accuracy of the result obtained in this study, two methods were considered, the Statistical analysis and the Ranking method. Random values of pressure from the PVT report were selected and the corresponding values of the isothermal compressibility coefficient obtained experimentally were used as the basis for evaluation. These values were compared to that obtained from this work and other existing charts.

##### 1) Statistical analysis:

The analyses used are the average relative error, average absolute error, standard deviation and coefficient of correlation.

##### 2) Average Relative Error (ARE), $E_r$

This is the deviation of an estimated value from a measured value and is defined by:

$$E_r = \frac{1}{n} \sum_{i=1}^n E_i \quad (10)$$

$$E_i = \frac{x_{exp} - x_{est}}{x_{exp}}, i=1, 2 \dots n \quad (11)$$

Where  $x_{exp}$  and  $x_{est}$  represent the experimental and estimated values respectively.  $E_r$  is an indication of the relative deviation from the experimental values. The lower the values of  $E_r$ , the more equally distributed are the errors between positive and negative values.

##### 3) Average Absolute Percent Error (AAE), $E_a$

The average absolute percent error is defined as

$$E_a = \frac{1}{n} \sum_{i=1}^n |E_i| \quad (12)$$

$E_a$  indicates the relative absolute deviation in percent from the experimental values. The smaller the value of error, the better the correlation.

##### 4) Standard Deviation, SD

The standard deviation of the data SD is a reflection of the dispersion of the data around the mean. It is expressed as the square root of the variance.

$$SD = \sqrt{\left(\frac{1}{n-1}\right) \sum_{i=1}^n (x_i - \bar{x})^2} \quad (13)$$

Where  $\bar{x}$  is the mean defined as:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad (14)$$

The lower the value of the standard deviation, the smaller the degree of dispersion.

##### 5) Correlation Coefficient

The correlation coefficient 'R' is the degree of success in reducing the standard deviation by regression analysis and it lies between 0 and 1. The value of 1 indicates a perfect correlation while 0 indicates no correlation among the given independent variables. Also, this implies that the larger the value of 'R', the greater the reduction in the sum of square errors and the stronger the relationship between the independent and dependent variables (Olajide and Ikiensikimama,2010).

$$R = \sqrt{1 - \frac{\sum_{i=1}^n [(X)_{exp} - (X)_{est}]^2}{\sum_{i=1}^n [(X)_{exp} - \bar{X}]^2}} \quad (15)$$

where

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n [(X)_{exp}] \quad (16)$$

$$\text{Min } Z_i = \sum_{j=1}^m S_{ij} q_{ij} \quad (17)$$

Subject to

$$\sum_{i=1}^n S_{ij} = 1 \quad \text{with } 0 \leq s_{ij} \leq 1 \quad (18)$$

Where

$s_{i,j}$  is the strength of the statistical parameter j of correlation i  
 $q_{i,j}$  is the statistical parameter corresponding to correlation i

$j = E_r, E_a, \dots, R'$  (where  $R' = 1-R$ )

$Z_i = \text{Rank (Rk)}$  of the desired correlation

$i =$  the number of correlation for a particular fluid property.

$$Z = 0.1(\text{ARE} \%) + 0.4(\text{AAE} \%) + 0.15(\text{SDr}) + 0.15(\text{SDa}) + 0.2(1 - R) \quad (19)$$

Where

ARE% = Average Relative Percentage Error

AAE% = Average Absolute Percentage Error

SDr = Standard Deviation of Average Relative Error

SDa = Standard Deviation of Average Absolute Error

R = Correlation Coefficient

### III. RESULT AND DISCUSSION

#### A. Isothermal Gas Compressibility Coefficient Computation

The composition data at the various depletion pressures in the valid reports were used in calculating the pseudo-reduced pressures and temperatures. The critical temperatures and pressures were corrected using Carr et al method for calculating these parameters. 290 data points were obtained for the pseudo-reduced pressures. The value of the pseudo reduced temperature was observed to have a range of 1.3 to 2.

The dimensionless isothermal gas compressibility coefficient equation defined by Ghedan et al using the differentiated Beggs and Brill z factor correlation was used in

obtaining the values of the coefficient at various pseudo reduced pressures and a constant pseudo-reduced temperature.

dimensionless isothermal gas compressibility coefficient in the Niger Delta region.

Table 1 show the range of data used for the analysis and the figure 1 below is the developed chart for finding the

TABLE I. RANGE OF DATA FOR STUDY

Parameter	Maximum Value	Minimum Value
Reservoir Pressure, Psia	7115	130
Reservoir Temperature, °F	274.6	151
Pseudo-reduced Pressure, P <sub>pr</sub>	10.80	0.09
Pseudo-reduced Temperature, T <sub>pr</sub>	2	1.3
Compressibility Factor, z	1.16	0.089

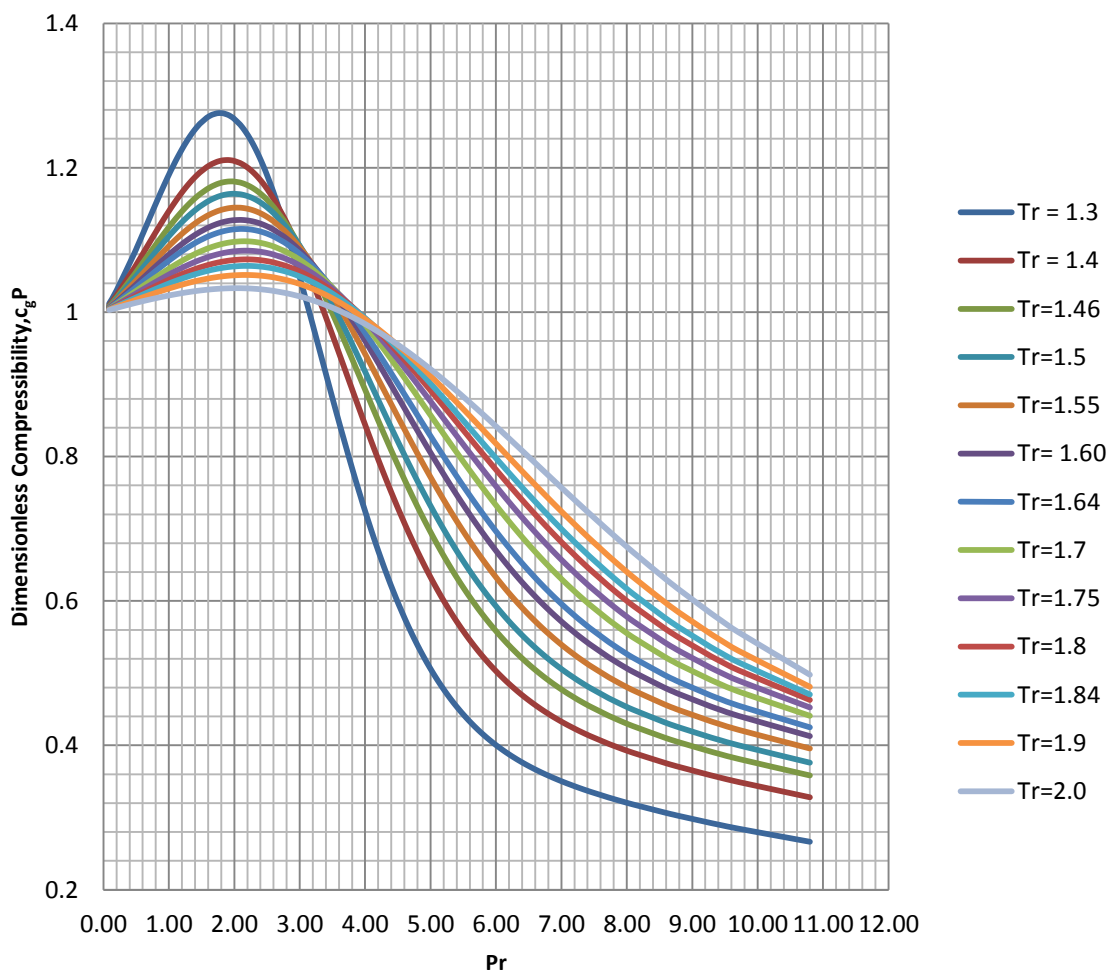


Figure 1. Dimensionless Compressibility as a Function of Reduced Temperature and Pressure (1.3 < Tr < 2.0; 0.09 < Pr < 11)

Also, the new procedure for calculating the dimensionless isothermal gas compressibility coefficient can be easily programmed using an excel spreadsheet. The z factor and the differential factor in the equation do not involve any form of

iteration to arrive at the desired value. Below is a stepwise process for obtaining the dimensionless isothermal gas compressibility coefficient.

*B. Spreadsheet Program for Calculating the Dimensionless Isothermal Gas Compressibility Coefficient.*

Step 1: Enter the value of the pseudo-reduced temperature and pressures to calculate the z factor.

Tr	1.5				
Pr	A	B	C	D	Z
0.20	0.417592462	0.057463	0.075651	0.968278	0.983353
0.85	0.417592462	0.281759	0.075651	0.968278	0.921673
1.35	0.417592462	0.493406	0.075651	0.968278	0.874601
1.81	0.417592462	0.712742	0.075651	0.968278	0.837291
2.30	0.417592462	0.982331	0.075651	0.968278	0.805131
2.61	0.417592462	1.169876	0.075651	0.968278	0.790097
2.98	0.417592462	1.404085	0.075651	0.968278	0.778113
3.53	0.417592462	1.796467	0.075651	0.968278	0.770943
3.90	0.417592462	2.076598	0.075651	0.968278	0.773025
4.39	0.417592462	2.484733	0.075651	0.968278	0.783271
4.87	0.417592462	2.904109	0.075651	0.968278	0.799622
5.32	0.417592462	3.333582	0.075651	0.968278	0.819977
5.84	0.417592462	3.860637	0.075651	0.968278	0.847563
6.24	0.417592462	4.286743	0.075651	0.968278	0.870772
6.69	0.417592462	4.794958	0.075651	0.968278	0.898646
7.36	0.417592462	5.60805	0.075651	0.968278	0.94247

Step 2: Using the z factors obtained, the pseudo-reduced temperature and pressures, the dimensionless isothermal compressibility coefficient is calculated

Tr	1.5						
Pr	z	H	I	J	K	$(\delta z/\delta Pr)_{Tr}$	$c_{gP}$
0.20	0.983353	-0.30137	0.582408	-0.05746	0.077095	-0.088623	1.01797
0.85	0.921673	-0.38749	0.582408	-0.28176	0.073628	-0.096636	1.089185
1.35	0.874601	-0.45402	0.582408	-0.49341	0.072551	-0.088891	1.137576
1.81	0.837291	-0.51395	0.582408	-0.71274	0.071889	-0.074871	1.161566
2.30	0.805131	-0.57918	0.582408	-0.98233	0.071341	-0.054964	1.157019
2.61	0.790097	-0.62053	0.582408	-1.16988	0.071053	-0.041128	1.136002
2.98	0.778113	-0.66859	0.582408	-1.40409	0.07076	-0.024871	1.095123
3.53	0.770943	-0.74215	0.582408	-1.79647	0.070377	-0.001324	1.006069
3.90	0.773025	-0.7905	0.582408	-2.0766	0.070157	0.0124441	0.937252
4.39	0.783271	-0.85607	0.582408	-2.48474	0.069891	0.0283358	0.841055
4.87	0.799622	-0.91858	0.582408	-2.90412	0.069665	0.0403492	0.754446
5.32	0.819977	-0.97846	0.582408	-3.33359	0.069469	0.0491445	0.681207
5.84	0.847563	-1.04729	0.582408	-3.86066	0.069263	0.0564212	0.611277
6.24	0.870772	-1.09979	0.582408	-4.28677	0.069119	0.0603121	0.568053
6.69	0.898646	-1.15931	0.582408	-4.795	0.068966	0.0633818	0.528414
7.36	0.94247	-1.24866	0.582408	-5.60813	0.068756	0.0660887	0.48378

C. Evaluation of Result:

TABLE II. DATA'S USED FOR THE STATISTICAL ANALYSIS

Pressure	Isothermal Compressibility (1/Psia)				
	Experimental (10 <sup>-6</sup> )	This Study (10 <sup>-6</sup> )	Ghedan et al (10 <sup>-6</sup> )	Trube's (10 <sup>-6</sup> )	Mattar et al (10 <sup>-6</sup> )
4800	113.38	111.78	118.75	136.06	134.53
5180	99.65	93.23	98.46	105.25	92.87
5690	89.94	77.71	70.3	79.36	80.92
6270	88.58	86.67	87.72	93.04	102.04
6481	72.36	73.07	70.98	80.26	80.26
7115	59.22	63.63	70.27	73.41	73.41

TABLE III. ERROR ANALYSIS

Statistical Parameters	This Study	Ghedan et al	Trube's	Mattar et al
Average Relative Error, E <sub>r</sub>	0.02529986	0.00418907	-0.08962322	-0.086492999
Average Absolute Percent Error, E <sub>a</sub>	5.33932393	8.21741438	12.8834541	14.26020706
Standard Deviation (relative error), SD <sub>r</sub>	0.07086104	0.13056779	0.12703224	0.13930031
Standard Deviation (absolute error), SD <sub>a</sub>	0.04866607	0.09468843	0.07653736	0.063082505
Correlation Coefficient, R	0.93963517	0.84153037	0.70140513	0.670624321
Rank	2.42	3.39	4.35	4.94

The first method used was the statistical method with Average Absolute percentage error of 5.34 and standard deviation of 0.071 and thus, agrees with literature that the best correlation should have the least Average Absolute percentage error. Also, the value of the correlation coefficient is close to 1 which indicates a near perfect correlation as compared to others.

The second method of choosing the best correlation which is the Ranking method combines all the statistical parameter into a single parameter called Rank. The correlation with the lowest rank is said to be the best. (Ikiensikimama, 2009).

It is observed from the table above that this study (work) has the lowest rank value of 2.42 as compared to that obtained by Ghedan et al, Mattar et al and Trube's work in finding the isothermal compressibility coefficient of natural gas.

IV. CONCLUSION

In this study, the PVT reports gathered were validated for accuracy. The valid reports were used for this analysis. The values of dimensionless isothermal gas compressibility coefficient were calculated using Ghedan et al correlation employing Beggs and Brill z factor correlation. The z factor correlation was differentiated to obtain the differential factor in the dimensionless isothermal gas compressibility coefficient equation. The obtained results was presented graphically for easy approximation and also evaluated using some selected statistical method and the ranking method by comparing the obtained results with other developed charts using the experimental values of isothermal gas compressibility coefficient as the basis. A good degree of accuracy was obtained between the developed correlation

and the experimental values which can be seen in the statistical and ranking analysis obtained.

The difference between the accuracy of the models tested can be inferred from the difference in techniques employed to obtain the z factor and the differential factor.

NOMENCLATURE

- ARE = Average relative error, %
- AAE = Average Absolute error, %
- E<sub>i</sub> = Relative deviation of estimated values from measured value, %
- E<sub>r</sub> = Relative Error, %
- M = Molecular weight, lb-mol
- M<sub>a</sub> = Apparent molecular weight,
- c<sub>g</sub>P = Dimensionless compressibility coefficient
- c<sub>g</sub> = Isothermal gas compressibility coefficient, 1/Psia
- P<sub>pr</sub> = Pseudo reduced pressure, psia
- Rk = Rank
- SD = Standard deviation
- SD<sub>a</sub> = Standard deviation of absolute error
- SD<sub>r</sub> = Standard deviation of relative error
- T = Temperature, °R
- T<sub>c</sub> = Critical temperature, °R
- T<sub>r</sub> = Reduced Temperature, °R
- T<sub>pr</sub> = Pseudo reduced temperature
- X<sub>est</sub> = estimated value
- X<sub>exp</sub> = experimental value
- y<sub>CO2</sub> = mole fraction of carbondioxide (CO<sub>2</sub>)
- y<sub>H2S</sub> = Mole fraction of hydrogen sulphide (H<sub>2</sub>S)
- y<sub>N2</sub> = Mole fraction of Nitrogen (N<sub>2</sub>)
- Z = Compressibility factor

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