Implementation of Random Modulator Per-Integrator Analog to Information Devices

Mahdi Safarpour\textsuperscript{1}, Mostafa Charmi\textsuperscript{2}, Siroos Toofan\textsuperscript{3}

\textsuperscript{1,2,3}Microelectronics Research Laboratory, University of Zanjan, Zanjan, Iran
(\textsuperscript{1}Mehdi.Safarpour@rocketmail.com)

Abstract—Recently Analog to information convertors (AIC) have been introduced as and alternative for conventional ADCs in some especial applications. In this paper an Analog to information convertor namely RMPI, is practically implemented. The recovery has been carried out through Compressive Sensing (CS) recovery algorithms. Through employment of an Nyquist rate ADC and an Microcontroller, random samples where acquired from the signal. On average the sampling rate reduced to 10\% to 50\% of the Nyquist sampling rate. The recovered signal demonstrates to be comparable with Nyquist rate acquired signal in terms of quality and dynamic range. Application of the sampler in practical application like wireless sensor networks and Tele monitoring systems, brings about the advantage of efficient usage of bandwidth for data transmission and also reduction in power consumption on the sensor level.

Keywords—Analog to digital, Wide Band Spectrum, Sparse Signals, Compressed Sensing, AIC

I. INTRODUCTION

We are living in an analog domain while trying hard to do all our signal processing in digital domain, and that is because the programmability, low cost and flexibly of the digital domain. The bridge between these two world is the Analog to digital convertor. ADCs are one the fundamental components of modern electronics systems. For example, consider the Cognitive Radio systems which has been designed to efficient utilization of the spectrum. The basis of its functionality is on the monitoring the spectrum and announcement of the frequency holes. A simple implementation of the system is through employment of an high rate ADC along with a digital FFT computer. This solution ends up in high power consumption and high cost of implementation [1]. On the other hand, CR signals have an interesting feature, which makes utilization of other methods promising. This feature is sparsity which means the occupied bandwidth in any given time is limited compared to the whole spectrum. Restrictions of using ADCs imposed by cost of implementation and power consumption and the idea of sparsity lead to invention of new signal acquisition devices called Analog to Information (AIC). Recently with development of Compressive sensing theory AIC has find practical application. There are various structures and architectures proposed to work as AIC, however we implement the Random Modulator Per-Integrator (RMPI) AIC. Compressive Sensing theory has been introduced as alternative way of sampling signals. The basis of CS is on random sampling and solving undetermined systems. Based on Nyquist Sampling theory for recovery of the signal the sampling rate must be at least twice high as the highest frequency of the signal [2]. However base on CS theory on the condition of sparsity of the signal in any domain, with a few measurement (much less than what Nyquist sampling might acquire) one can acquire and recover sparse signals.

II. MATERIAL AND METHODS

A. Compressive Sensing

Based on CS theory providing that the signal is sparse in some domain, one can acquired and recover signals with fewer samples than that of the Nyquist sampling rate captures. On the other hand most of signals in nature are sparse. To name a few we can mention images, MRI, radar, etc. The point is that the signal does not need to be sparse in time or space but it might be sparse in a transform domain like Fourier, Wavelets, Discrete Cosines Transform and many other[3]. For example consider the sparse representation of the signal shown in Figure 1.a in the Figure 2.b.

As it is observed in Fig.1 the signal in transform domain contains only a few non-zero coefficients while it is not sparse in time domain at all. Remember, indeed on the contrary to the conventional Nyquist sampling in CS samples must be taken randomly not uniformly.
Assume X as an sparse signal with length of N in the Ψ domain where there are k (k<<N) non-zero coefficients, under the below system is measured and compressed to the Y with M rows (M<<N):

\[ Y_{M \times 1} = \Phi_{M \times N} \Psi_{N \times N} X_{N \times 1} \]  

(1)

The operation is shown in Figure 2 as below.

Figure 1. a) signal in time domain, b) sparse representation of a in DCT domain

Obviously this cannot be solved through conventional algebraic methods, because this system is underdetermined system and there are infinitely solutions exits for these types of systems. For recovery of the X under this system through CS theory two conditions must be satisfied, first Φ matrix must be incoherent with Ψ matrix, in other words the multiplication of these two must not result in a matrix which one of the columns has more weights than others. In other word the length of the signal in two domain of time and transform must be the same [3,4].

\[ \|X_1 - X_2\|_2 \approx \|Y_1 - Y_2\|_2 \]  

(2)

This condition is satisfied through selecting elements of the Φ matrix randomly. The \( L_1 \) minimization has been proposed for recovery of the X as below:

\[ \hat{X} = \arg \min_{\|X\|_1} \]  

s.t \( Y = \Phi X \)  

(3)

The above equation is in fact an optimization problem which can easily be solved through linear programming.

B. Random Modulator Per-Integrator

The Random Modulator Per-Integrator (RMPI) might be better understood in the limited discrete and non-continues domain [5-8]. Imaging the set of band-limited signals, which we will approximate by 1-D matrices of large dimension very accurately in the Fourier lattice. Regularly, ADCs take cyclic samples, which are represented in the discrete context by a diagonal identity matrix. The Random Modulator Per Integrator, on the other hand, employs a matrix which is not diagonal, and its energy is extended in all of the matrix elements [8]. Particularly, each element is randomly allocated either +1 or -1. The advantage of this method is that CS theory permits us to take much less samples than regular approaches do. The factual RMPI is a bit more complicated, it is complicated because of the fact that making an desired large matrix with all +1 and -1 element in hardware is hard and costly. In place of the huge matrix is form by sections of +/− 1 vectors Multiple channel working sequentially are employed to form such matrices. There are also four channels, which allows these sub-blocks to be larger, and therefore the matrix more resembles almost exactly a full +/− 1 matrix. Figure 3 show the conceptual diagram of RMPI AIC [7].
The system of equations formed by the RMPI is as below [9]:

\[ y_{NUS} = S_{M \times N} x_{N \times 1} + z_{M \times 1} \quad (4) \]

Remember that the samples are not taken from any desired point on the grid \([0,T]\) but they are chosen from the grid \(t_1, t_2, t_3, \ldots, T\). AIC is a very simple system for implementation. It is sufficient to implement an ADC with random mixing of the signal taken driven by a the clock into our work, and using a digital processor and retrieval algorithms of compressed sensing, to recover the original signal and to obtain digitized version of it. Remember the speed of the sampling of RMPI must be high as the rate must be equal still Nyquist rate because samples may also be selected at random samples. The reader may question about the advantage of RMPI over the ADC, to answer it must be said that although both have the same sampling rate, the average sampling rate of RMPI is much lower than the ADC sampling, so RMPI consumes less power than the ADC. On the other hand in Tele monitoring and wireless sensor networks bandwidth is very important. Through employment of RMPI data can be compressed at the sensor level. This in turn leads to reduced power consumption.

### III. SIMULATION RESULTS

As it has stated in the previous section, implementation of RMPI is feasible through random sampling and mixing and integrating the signal then using a low rate ADC quantizing and recovering the signal. We chose a four channel 1 MHz RMPI structure for simulation and implementation. The circuit has been simulated in ProSpice and an AIC have been implemented in practice through a random sampling. For recovery OMP [5] algorithm was chosen which is an greedy sparse recovery algorithm. Figure 4 show an sparse signal (consisting of a single sinusoid ) along with its reconstructed counterpart through AIC. Simulation results are plotted in Figure 4 shows the results for the sampling. Root-mean-square error has been adopted as an quality criteria. The RMSE is calculated as below:

\[ \text{RMSE} = \sqrt{\frac{\sum_{k=1}^{n} (Y_{\text{recovered}} - Y_{\text{original}})^2}{n}} \quad (4) \]

Where \(Y_{\text{recovered}}\) is the reconstructed signal and \(Y_{\text{original}}\) is the original signal which is sub-sampled.

As it is seen in Figure 5 the quality of the recovered signal decreases with increase in the input noise level. This is a natural result because with more noise based on CS theory there will be less sparseness. Also a comparison between three different signal recovery and acquisition technique namely \(L1\) min. recovery, greedy recovery and ADC system acquisition were done which is depicted in figure below.
IV. CONCLUSION

In this paper the concept of compressive sensing theory and analog to information convertors has been introduced. Among all architectures for AICs we introduce RMPI analog to information for implementation. The RMPI AIC has been simulated and results presented. Based on the results the RMPI can successfully recover sparse signals from incomplete set of observation of the signal. Simulation results suggest an great compression of the signal at the acquisition level. Therefore the application of the AIC is not limited to acquisition of the signal only, but also it can be used for compression of the data while receiving them which makes them an perfect choice for environments with restricted power and bandwidth.

V. REFERENCES