

# An Improved Transmission Equation under Environmental Influences

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**Abstract-** Radio frequency (RF) communication channel severely suffers from tropospheric scintillation fading caused by the dynamic nature of the atmospheric conditions thereby impairing its performance and availability; this induced channel fading effect must be accounted for in the link transmission equation. In this paper, we have proposed an improved transmission link equation by taking into account scintillation fading effect and magnitude of the refractive-index structure parameter that play very important role in links calculations. This transmission model provides the basis for communication engineers a platform to work with in the link budgetary for planning and design of low margin systems of free space communication link.

**Keywords-** Transmission Equation, Scintillation fading effects, Communication Link, Link budgetary and planning

## I. INTRODUCTION

During radiowave propagation an interaction between waves and environment attenuates the signal level; causes path loss and finally limits coverage area (Famoriji and Olasoji, 2013). Radio frequency wireless transmission performance is highly vulnerable to adverse dynamic atmospheric conditions, which is unstable at the tropospheric layer of the atmosphere where it's been accessed. Kolawole (2003) reported that the troposphere contains 99% of the water vapour whose concentrations vary with latitudinal position in the atmosphere. Atmospheric scintillation occurs as a result of the variations in the refractive index due to inhomogeneities in temperature and pressure changes (Tsiftsis et al., 2009). This results in rapid fluctuations of the amplitude and phase of the wave front at the received signal, i.e. known as fading or scintillation, impairing the system performance and reliability particularly for link ranges of 1 km and above.

Over the years, some tropospheric scintillation models e.g. Karasawa model, Otung model, Van De Kamp model, Ortgies-N and Ortgies-T models, ITU-R model etc. have been developed that account for this fading effect (Nadirah et al., 2012). Research studies continue, though, in predicting tropospheric scintillation both theoretical and empirical. Regardless of the methods employed, inclusion of the main link parameters (for example, the frequency, elevation angle and antenna diameter) and meteorological data (for instance,

the humidity at ground level and mean temperature) are needed in order to obtain reliable scintillation prediction. Tropospheric scintillation is therefore a signal propagation impediment, which must be accounted for in order to complete the link budget for design of low margin systems.

Pt/Pr relationship is a fundamental expression in antenna propagation technique (Kolawole, 2014). Friis antenna and propagation equation is an essential tool in the design and analysis of wireless communication system, which relates the power fed Pt to the transmitting antenna and the power received Pr by the receiving antenna when the two antennas are separated by a sufficiently large distance R; i.e.,

$$R \gg \frac{2D_{\max}^2}{\lambda} \quad (1)$$

where, at far zones, Dmax is the maximum antenna aperture diameter (m), and  $\lambda$  is wavelength (m). However, antenna and propagation model that factors in environmental influences (tropospheric scintillation effect) requires consideration.

## II. ESTIMATION OR PREDICTION METHODS

Several works on tropospheric scintillation have been done both theoretically and experimentally Uysal *et al* (2006); Yoshio and Yamada, (1988); Mouldsley and Vilar, (1982), and there are different kinds of already proposed scintillation methods Dissanayake (2002) and Tatarskii (1961). Among all the methods is the Tatarskii theory which is based on atmospheric turbulence theory, also the CCIR model (CCIR, 1986) was based on one year measurement can yield useful estimates of scintillation fading. However, both methods have their own drawbacks in practical application to satellite system design. For example, in the Tatarskii model, estimating an appropriate figure of a parameter for the fine structure of atmospheric turbulence along a given transmission path is not usually feasible, while the CCIR model also does not incorporate the meteorological parameters which are proven to have a major effect. Uysal *et.al* (2006) developed an error performance bounds for coded FSO systems to mitigate turbulence-induced fading over atmospheric channels. Kramer and Goodman, (2001) observed the scintillation in the MAGR/S that occurs because the multiple available paths of propagation though are geometrically far apart and rapidly

varying. Tatarskii according to Strangeways *et, al* (2007) introduced a theoretical formulation for the estimate of log-amplitude fluctuations based on the assumption that the spatial structure of the atmospheric refractive index. When the atmospheric turbulence lies in the inertial sub-range where a relation:  $R_{min}^2/\lambda \ll l \ll R_{max}^2/\lambda$  is satisfied, Where:  $R_{min}, R_{max}$  is the Inner, outer scale of refractive index irregularities;  $l$  is the effective path length between the boundary of turbulence and the reception point, is the wavelength of the radio wave; the variance of log-amplitude in dB is given by:

$$\sigma_x = 42.9 \left(\frac{2\pi}{\lambda}\right)^{7/6} \int_0^L C_n^2(r) r^{5/6} dr \quad (dB) \quad (2)$$

The structure parameter of the refractive index  $C_n$  in Equation (2) is varying along the earth-space path.  $C_n$  - Parameter depends not only on the variance of the atmospheric refractive index but also on the outer scale  $R_{min}$  of irregularities. Therefore, it is difficult to identify values of such parameters along a given slant path directly from meteorological parameters which are generally available, and consequently remains difficult to calculate.

However, a more practical model was proposed by the CCIR (Kramer and Goodman, 2001) based on measured data at a frequency of 7.3GHz with an elevation angle of  $1^\circ$  and an antenna diameter of 10m together with theoretical scaling for the frequency, elevation angle, and antenna dish diameter. Since the CCIR model is made-up of simple factors or terms and its known to provide estimated values close to the measured data reported thus far, it could be a good method for tropospheric scintillation prediction at present. However, Hussein, (2009) reported that the model may not be appropriate to extrapolate from the result at a  $1^\circ$  elevation angle, which is the most important range when designing satellite communication systems. Also, parameters representing meteorological elements are not included in the equation, therefore regional and seasonal dependence cannot be explained. The estimated value merely provides the standard deviation of the fluctuations and does not cover the estimate of signal fading as a function of time percentage. This paper attempted the extension of the model for use in free space optical communication system.

#### A. Experimental Data Piloting the Scintillation Model Considered

The scintillation measurement (Kramer and Goodman, 2001) was used as a follow-up for constructing the new prediction model. The measured data were obtained in the 11/14 GHz low elevation angle propagation experiment for one year at the Yamaguchi satellite. During the course of 4 year measurement at Yamaguchi, long term data for elevation angle of  $4^\circ$  (satellite:  $57^\circ$  E),  $6.5^\circ$  ( $60^\circ$  E) and  $9^\circ$  ( $63^\circ$  E) the power of csec $\Theta$  was about 1.3.

The probability density function of amplitude variation (in dB) during relatively short periods of time (approximately one hour) shows a Gaussian distribution including fairly large scintillation whose rms fluctuations are up to 1.6dB. Also, Distribution of the standard deviation itself for long term

variations over a month can be closely approximated with a gamma distribution. Likewise, there is not a very large deviation from the gamma distribution for the yearly basis.

This indicates that each of the two parameters essential for the determination of the gamma distribution is in fact directly determined by the other. Based on the properties just stated the following equation can be derived (Hussein, 2009):

$$x(m, p) = m \times \eta(p) \quad (3)$$

where  $x$  is the signal level for  $m$  mean value of rms fluctuations in dB and  $P$  percent of the time such that relation between  $\eta$  and  $P$  is as following:

$$\eta(p) = 1.099 \times 10^4 \iint_{\eta_0}^{\infty} U^8 \exp\left(-\frac{v^2}{(2U^2-10U)du dv}\right) \quad (4)$$

The time percentage factor  $\eta$  could be obtained as a function of the time percentage  $P$  from numerical calculations.

Fig. 1 shows three kinds of cumulative time distributions of signal level variation obtained from a one month measurement (Solid lines) together with calculated curves using eqn. (3) (dotted line). As it is clear from the figure, measured and calculated curves where the signal level is enhanced (namely,  $P$  of 50 to 99.99 percent) have a fairly good coincidence. While a noticeable discrepancy exists in the signal fade region, particularly for  $P$  of less than one percent. This may be due to the asymmetry of signal level variation between fading and enhancement. Fig. 2 shows time percentage factor  $\eta$  (namely  $x$  normalized by  $m$ ) for both signal fading and enhancement of the data shown in Fig.1. The best fit curves for both fading and enhancement are represented by dotted and solid lines, respectively. The solid line in the figure is the same as the one obtained theoretically in Equation (3).

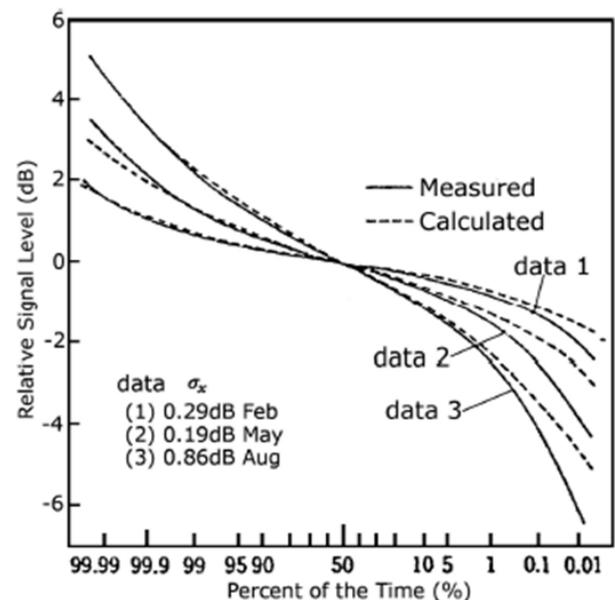


Figure 1. Cumulative Time Distribution of Long Term Signal Level Variation due Scintillation (Source: Yoshio and Yamada, 1988)

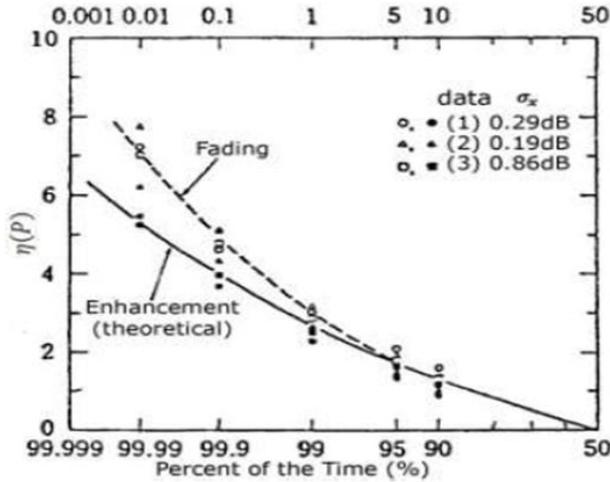


Figure 2.  $\eta$  factor versus time percentage  $p$  (Source: Yoshio and Yamada, 1988)

In this paper, the tropospheric scintillation index  $\sigma_x$  developed by CCIR is used (Hussein, (2009)); specifically,

$$\sigma_x = (\sigma_{xref} \times \eta_f \times \eta_{\square} \times \eta_{Da}) \quad (dB) \quad (5)$$

$$\sigma_x = (0.0228\sigma_{xref} \times f^{0.45} \times cosec^{1.3}(\theta) \times \sqrt{Q(Da)}) \quad (dB) \quad (6)$$

Where; the normalized or reference standard deviation  $\sigma_{xref}$  is given by

$$\sigma_{xref} = 0.15 + 5.2 \times 10^{-3} N_f \quad (7)$$

$$N_f = N_{wet} = \frac{3730Ue_s}{(t+273)^2} \quad (8)$$

$$e_s = 6.11 \exp\left(\frac{19.7t}{t+273}\right) \quad (9)$$

Where  $t$  is the average temperature ( $^{\circ}C$ ),  $U$  is the relative humidity (%),  $e_s$  is the saturated vapor pressure (mb). It is applicable in the temperature range of  $-20^{\circ}C \leq t \leq 50^{\circ}C$  where  $T$  is the measured temperature (in K) and  $U$  is the relative humidity (in %). However, for completeness, in desert or semi-desert areas—constituting part of Nigeria—fog (or rain) is seldom. For these cases Radio frequency communication systems can be deployed over long distances and the scintillation effect has to be taken into consideration, so refractivity in Eqn. (8) is replaced with  $N_{dry}$ , (Olasoji and Kolawole, 2010):

$$N_f = N_{dry} = 77.6848 \frac{P}{T} \quad (10)$$

This is valid for frequency up to 30 GHz and for normally encountered ranges of pressure,  $P$ , temperature,  $T$  and humidity,  $U$ . The water vapor pressure is given specifically as:

$$P = \frac{Ue_s}{100} \quad (hPa) \quad (11)$$

$$\eta_f = \left(\frac{f}{11.5}\right)^{0.45} \quad (12)$$

$f$  = frequency (GHz);

$$\eta_{\square} = \left(\frac{cosec\theta}{cosec6.5}\right)^{1.3} \quad (13)$$

or

$$\eta_{\square} = \frac{2h}{\sin\theta + \sqrt{\sin^2\theta + \frac{2h}{R_e}}} \quad (14)$$

Where;

$h$  is the effective height of the water vapor in atmosphere (2km in this model)

$R_e$  is effective earth radius (8500km); this varies from locations to locations (6378km for Nigeria)

$\theta$  is elevation angle (degree)

$$\eta_{Da} = \sqrt{Q(R)/Q(7.6)} \quad (15)$$

Where the antenna diameter dependent factor  $Q(R)$  is given by Uysal *et, al* (2006) from a piecewise linear approximation of the antenna aperture averaging factor is given as:

$$Q(R) = 1 - 1.4(R/\sqrt{\lambda L}) \quad \text{for } 0.0 \leq \left(\frac{R}{\sqrt{\lambda L}}\right) \leq 0.5$$

$$Q(R) = 0.5 - 0.4(R/\sqrt{\lambda L}) \quad \text{for } 0.5 \leq \left(\frac{R}{\sqrt{\lambda L}}\right) \leq 1$$

$$Q(R) = 0.1 \quad \text{for } 0.0 \leq \left(\frac{R}{\sqrt{\lambda L}}\right) \leq 0.5$$

where;

$R$  is the effective radius of circular antenna aperture (m) given by:

$$R = 0.75 \left(\frac{Da}{2}\right) \quad (16)$$

$Da$  is the diameter of the reflector (m),  $\lambda$  is the operating wavelength (m),  $L$  is the slant distance to height of a horizontal thin turbulent layer ( $=\eta_{\square}$ ).

Eqn. 5 is for the case when elevation angle is  $\theta \leq 5^{\circ}$  while eqn. 6 is applicable when elevation angle is greater than five degree ( $\theta > 5^{\circ}$ ). However, based on the study of Hussein, (2009); at a certain frequency value, the magnitude of scintillation (rms fluctuation) would be obviously decreased with increasing of elevation angle. This paper therefore considered the second case of the model (i. e  $\theta > 5^{\circ}$ ).

Technological progress can be useful only if accurate quantitative measurements prove to be feasible through atmospheric turbulence. Several boundary layers-namely planetary, the surface and the internal or constant flux-are pertinent to the study of atmospheric turbulence. The surface layer is the lowest layer of the atmosphere (and troposphere-the region of interest). It is the layer, where air is in contact with the surface and where strong vertical gradients in temperature, humidity, wind and scalars exist. The layer is between 1 and 2 km of the atmosphere.

### III. PROPOSED MODIFICATION TO FRIIS TRANSMISSION EQUATION

The main disadvantage of Friis' method is the need to incorporate the two simultaneously changing parameters causing fluctuation in the received power, namely scintillation index  $\sigma_x$  and polarization loss factor,  $L_{pf}$ . In this paper, unlike other works, the changing parameters are incorporated in the Friis links transmission equation thereby broadening its practical applications.

Friis transmission equation gives amount of power an antenna received under ideal conditions from another antenna. Using Fig. 3 as a guide, the Friis transmission equation works under certain conditions:

- antennas must be in radiating far-field, i.e.  $R$ -given by Eqn. (1), or  $R > 10\lambda$  for small antenna (Kolawole, 2013) where  $D$  and  $\lambda$  are as previously defined;
- antennas are in unobstructed free space i.e. line of sight transmission mode which is the same as that of FSO;
- bandwidth is narrow enough that a single wavelength can be assumed and antennas are correctly aligned and polarized (Nikolova, 2012).

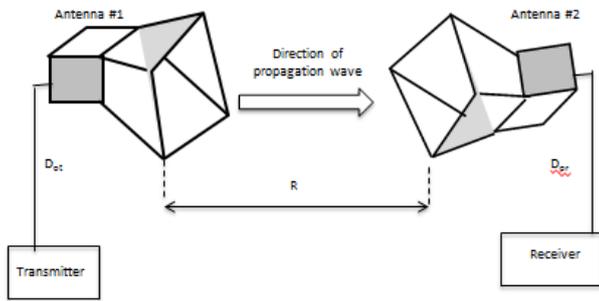


Figure 3. Two antennas separated by distance  $R$ . (Source: Kolawole, 2014)

Suppose the transmitting and receiving antennas are not matched to their respective lines or loads (reflection efficiencies are not unity) and the polarization of the receiving antenna is not polarization-matched to the impinging wave (polarization loss factor and polarization efficiency are not unity), then the power ratio  $P_r/P_t$  expression can be written as (Kolawole, 2014):

$$\frac{P_r}{P_t} = \eta_{or} \eta_{ot} (1 - |\Gamma_t|^2) (1 - |\Gamma_r|^2) \frac{d_{or} d_{ot}}{4\rho R^2} |L_{PF}|^2 \quad (17)$$

where  $\eta_{or}$  and  $\eta_{ot}$  are receiving and transmitting antenna efficiency, respectively; and  $\Gamma_r$  and  $\Gamma_t$  are voltage reflection coefficient at the input terminals of the receiving and transmitting antenna, respectively, which are related to their respective impedances; and  $d_{or}$  and  $d_{ot}$  are receiving and transmitting antenna directivity, respectively. In reality, directivity and polarization loss factor are direction dependent, i.e. azimuth  $\phi$  and incident  $\theta$ . However, if reflection and

polarization-matched antennas aligned for maximum directional radiation and reception, then Eqn. (17) reduces to:

$$\frac{P_r}{P_t} = \frac{d_{or} d_{ot}}{4\rho R^2} G_{or} G_{ot} \quad (18)$$

the well-known Friis Transmission Equation, where  $G_{or}$  and  $G_{ot}$  are receiving and transmitting antenna gain, respectively, defined as  $G_{or} = \eta_{or} d_{or}$ , and  $G_{ot} = \eta_{ot} d_{ot}$ . The term  $(\cdot)^2$  in Eqn. (18) is called the free-space loss factor, and it takes into account the losses due to the spherical spreading of the energy by the antenna. Incorporating Eqn. (6) in Eqn. (17), the Friis Transmission Equation is modified to account for factors in environmental influences and polarization losses suitable for FSO communication link. Specifically:

$$\frac{P_r}{P_t} = 10^{0.1\sigma_x} \eta_{or} \eta_{ot} (1 - |\Gamma_t|^2) (1 - |\Gamma_r|^2) \left(\frac{\lambda}{4\pi R}\right)^2 d_{or} d_{ot} |L_{PF}|^2 \quad (19)$$

Bearing in mind that the unit of the rms fluctuation or fading due to scintillation ( $\sigma$ -as in Eqn. (6)-is in decibel (dB) whose absolute value is  $10^{0.1\sigma_x}$ .

### IV. SIMULATION

In order to validate our modifications to transmission equation for communication link, we first simulate ideal situation without environmental effects, and second with the scintillation index. The results presented here give an idea of the reliability of free space communication ensured by a particular FSO deployed at various distances.

The antenna efficiency was assumed to be unity though this may not exist in practice. Effective height ( $h$ ) of water vapor in atmosphere was taken to be 2km. Elevation angle  $\theta$  considered is  $20^\circ$ . The temperature  $t$  and relative humidity  $U$  were set at  $37.1^\circ\text{C}$  and  $24\%$ , respectively, while the corresponding pressure  $P$  using eqn.(11) gave 333.62 (hPa) while the antenna aperture diameter was taken to be 10m. Also, the transmitted power was set at 500kW, frequency at 14GHz, transmission distance between 0 and 5km while the transmitting and receiving antenna gains were set at 34000 and 32000, respectively and the original effective earth radius is 6378km which is for Nigeria under consideration. It was assumed that reflection and polarization-matched antennas are aligned for maximum directional radiation and reception. The simulation was carried out using a Matlab software package (R2011a).

Figure 4 show the results of simulations for the transmission equation without and with scintillation fading effect for wet refractivity, dry refractivity and dry-wet refractivity. The variation in amplitudes is an indication of the effect of atmospheric scintillation causing fluctuation in the received power.

As seen from Fig. 5 the scintillation fading, aperture averaging, and magnitude of the refractive-index structure parameter play very important role in wireless links calculations. It could also be deduced that the wireless link is

greatly impaired by foggy environment which will consequently limit the coverage area. Also, it depicts a transitory sudden situation, which may occur in real life and affect the link. The wet term contributes to the major variation of the total value of the refractivity because the scintillation intensity strongly depends on it. The term  $N_{dry}$  is proportional to the density of the gas molecules in the atmosphere and changes with their distribution but its relatively stable. The proposed model can therefore be used to estimate the availability and reliability of radio link design in a particular environment.

## V. CONCLUSION

In this paper, we have improved the transmission link equation by taking into account two simultaneously changing parameters affecting the reliable free space communication. Free space communication's channel suffers severely from fading caused by tropospheric scintillation index, thereby limiting its deplorability in many cases. The proposed transmission model provides the basis for communication engineers a platform to work with, in the link budgetary for planning and design of low margin systems of free space communication link.

## REFERENCE

- [1] Famoriji J. O. and Olasoji Y. O., (2013) , "Radio Frequency Propagation Mechanism and Empirical Model for Hilly Areas", Canadian Journal of Electrical and Electronics Engineering, Vol. 4, No. 2, pp 65-70.
- [2] Kolawole, M.O. (2003). "Radar systems, peak detection and tracking", Oxford: Elsevier Science.
- [3] Tsiftsis T. A., Sandalidis G. H., Karagianids G. K. and Uysal M. (2009), "Optical Wireless Links with Spatial Diversity over Strong Atmospheric Turbulence Channels" IEEE Transactions on Wireless Communications Vol. 8, No. 2.
- [4] Nadirah, B. A. R., Rafiqul, I., Mandeep, J. S. and Hassan D. (2012), "Comparison of Tropospheric Scintillation Models on Earth-Space Paths in Tropical Region", Research Journal of Applied Sciences, Engineering and Technology, 4(11), pp. 1616-1623.
- [5] Kolawole. M. O. (2014). "Antenna System Design and Theory". ARCSTE-E/FUTA Lecture note
- [6] Uysal, M.; Li, J.; Yu, M., (2006) "Error Rate Performance Analysis of Coded Free-Space Optical Links over Gamma-Gamma Atmospheric Turbulence Channels," IEEE Transactions on Wireless Communications, Vol. 5, No. 6.
- [7] Yoshio, K.; Yamada, M., (1988) "Tropospheric Scintillation in 11/14 GHz Bands on Earth-Space Paths with Low Elevation Angles," IEEE Trans. Antenna Propagation, vol.36, No.4 PP. 563-569.
- [8] Mousley, T.J. ; E.Vilar.,(1982) "Experimental and Theoretical States of Microwave Amplitude Scintillation on Satellite Down-Links" IEEE Tran. Antenna Propagation, Vol. AP.30, No.6, PP.1099-1106.
- [9] Dissanayake, A., (2002) "Ka-Band Propagation Modeling for Fixed Satellite Applications," Journal of Space Communication Issue No. 2.
- [10] Tatarskii, V. I., (1961) "Wave Propagation in a Turbulent Medium" New York.
- [11] CCIR, (1986) "Effects of Tropospheric Refraction on Radio-Wave Propagation" CCIR SG-5. Rep. 718-2, ITU, Geneva.
- [12] Kramer, L.; Goodman, J. L., (2001) "Global Scale Observations of Ionospheric Instabilities from GPS in Low Earth Orbit," American Institute of Aeronautics and Astronautics, Inc.,
- [13] Strangeways, H.J.; Gherm, V.E.; Zernov, N.N., (2007) "Modeling and mitigation of the effect of scintillations on GPS", ELMAR 2007 Volume
- [14] Hussein M. A., (2009), "Scintillation Effect on Satellite Communications within Standard Atmosphere", Anbar Journal of Engineering Sciences, Vol. 2, No. 2, PP-17-27.
- [15] Olasoji, Y.O. and Kolawole, M.O. (2010). "Signal Strength Dependence on Atmospheric Particulates," International Journal of Electronics and Communication Engineering, 4(3): 283-286
- [16] Kolawole. M. O. (2013). "Satellite Communication Engineering," 2nd Edition, NY: CRC Press
- [17] Nikolova, N. (2012), "Friis Transmission Equation and Radar Range Equation" Lecture Note 6.

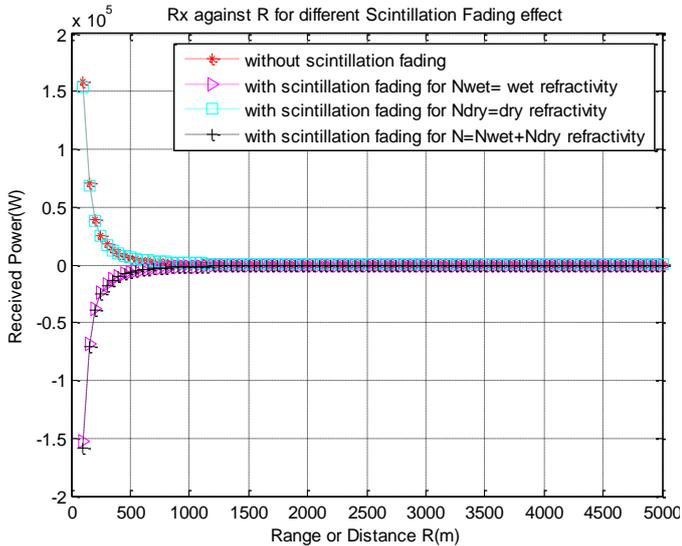


Figure 4. Plot of Power Received against Range without and with Scintillation Fading Effect for Wet, Dry and Wet-Dry Refractivity

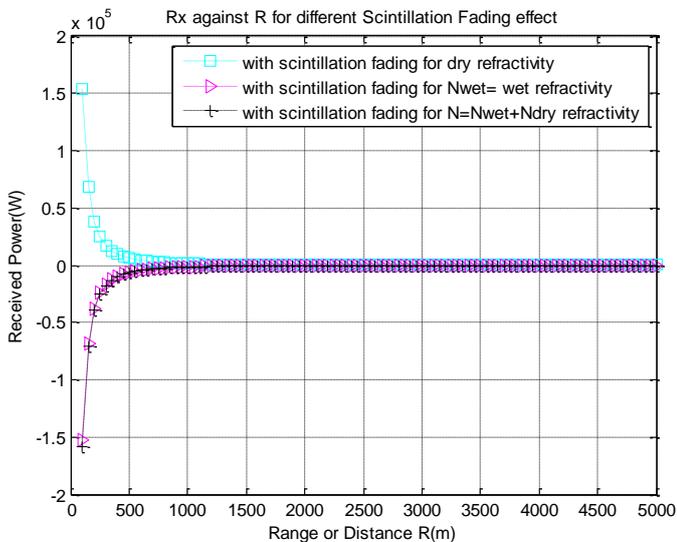


Figure 5. Plot of Power Received against Range with Scintillation Fading Effect for Wet, Dry and Wet-Dry Refractivity