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An Analysis of Steady Convective MHD Fluid Flow in Parallel Vertical Semi-Infinite Plates with Constant Magnetic Field

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Abstract- An investigation of MHD stokes free convection of an incompressible electrically conducting fluid between two vertical parallel semi-infinite plates subjected to a homogenous magnetic field has been done. The effect of a uniform magnetic field directed perpendicularly to the plates on the dynamic behavior of the fluid is studied. One plate is rigid, porous and fixed along the y axis while the other plate is placed at a distance h from the y axis and impulsively started in the flow direction. Further, a study on how Prandtl number and Hartman Number affect velocity and temperature profiles is carried out. The velocity profiles and temperature distribution are governed by a coupled set of continuity, momentum and energy equations. The generated equations have been solved numerically by the central finite difference approximations. The results obtained are discussed and presented both in tabular and graphical form. An increase in Hartmann is found to cause a decrease in velocity profile while an increase in Prandtl leads to a fall in temperature distribution. These results are found to merge with the physical situation of the flow.

Keywords- *porous plate, incompressible, magneto hydrodynamics*

I. INTRODUCTION

A fluid is a substance that can flow in an enclosure, in a pipe, in a channel or over a plate. Heat is transferred from one point to another as fluids flow. Heat transfer in fluids is called convection. In engineering devices, fluid flows occur in the presence of magnetic field. Fluid flow in the presence a magnetic field is called hydro magnetic flow and the study of hydro magnetic flows is called magneto hydro dynamics (MHD).A steady flow of an incompressible, viscous and electrically conducting fluid is very significant due to its application in MHD generators, pumps and flow meters. Further, this flow in a porous media is used to study the migration of underground water, movement of oil, gas and water through the reservoir, water purification, ceramic engineering and powder metallurgy.

II. LITERATURE REVIEW

The history of MHD flow traces back to 1930's.Since then a lot of research has been done on this area and considerable progress made. Some of the recent investigations include the influence of lateral mass flux on the free convection flow past a vertical flat plate embedded in a saturated porous medium done by [1] unsteady heat transfer to pulsatile flow of a dusty viscous incompressible fluid in a channel was investigated by[2] . [3] Solved magneto hydrodynamics stokes problem of convection flow for a vertical infinite plate in a dissipative rotating fluid with Hall current. This is an analysis of the effects of various parameters on the concentration velocity and temperature profiles. [4]conducted a study on Radiation and free convection flow past a moving plate while[5] presented their work on MHD free convection heat and mass transfer of a heat generating fluid past an impulsively started infinite vertical plate with Hall current and radiation absorption a solution to hydro magnetic flow of dusty visco- elastic fluid between two infinite parallel plates was done by[6].Further,[7] presented their work on MHD stokes free convection past an infinite vertical porous plate subjected to a constant heat flux with ion-slip and radiation absorption. They discussed their tabulated results on concentration, velocity profiles and temperature distributions both theoretically and graphically whilst [8] studied a steady MHD flow of an electrically conducting fluid between two parallel infinite plates when the upper plate is made to move with constant velocity while the lower plate is stationary. [9] Investigated Radiation effects on MHD couette flow with heat transfer between two parallel plates whereas [10] carried out an investigation on Free Convection MHD Flow with Thermal Radiation from an Impulsively Started Vertical Plate. They established that velocity increases with a decrease in magnetic field parameter. In addition they realized that dimensionless temperature decreases with an increase in thermal radiation. [11] Carried out an investigation on hydro magnetic free convectional currents effects on boundary layer thickness while [12] carried out an investigation on the effects of Hall current and Rotational Parameter on dissipative fluid past a vertical semi-infinite plate. They found that an increase in Hall parameter for both cooling and heating of the plate by free convection currents has no effect on temperature profiles but leads to an increase in velocity profiles. [13] Conducted an investigation on unsteady transient free convection MHD flow between two long vertical parallel plates with constant temperature and variable mass diffusion. They established that velocity and skin friction of the fluid increase with increase with the value of time but decrease with increasing the value of the Prandtl number, Schmidt number and magnetic parameter.

Similarly they found that an increase in Rotational parameter led to a decrease in velocity profile when the Eckert number was 0.01 and an increase in velocity profile when Eckert number was 0.02. Furthermore they realized that an increase in time led to an increase in both primary and secondary velocity profiles in case of cooling of the plates by convection currents but led to a decrease in velocity profiles in case of heating the plates by convection currents. More over [14] investigated on Steady MHD Poiseuille flow between two infinite porous plates in an inclined magnetic field. They established that a high Hartmann flow (high magnetic field strength) decreases velocity. In addition [15] carried an investigation on unsteady free convection MHD flow and heat transfer between two heated vertical plates with heat source. It was found that an increase in Hartmann number caused high velocity profiles near the walls and low velocity profiles at the center between the walls.

In view of the above studies, a steady flow in semi-infinite vertical plates with one plate porous and rigidly fixed along the y axis and the other placed at a distance h has not received adequate attention. Hence, this study has been carried out to analyze the effect of Hartmann and Prandtl numbers on velocity profiles and temperature distribution on a steady flow in semi-infinite porous plates.

III. GEOMETRY OF THE PROBLEM

An electrically conducting, viscous and incompressible fluid is made to flow between two vertical parallel semi-infinite plates. One plate is rigid, porous and fixed along the y axis while the other plate is placed at a distance h from the y axis and is impulsively started at constant velocity U in the flow direction. A uniform magnetic field is then applied normal to the plates in the positive x-direction. It is assumed that the magnetic Reynolds number is so small such that the induced magnetic field is neglected in comparison to the applied one.

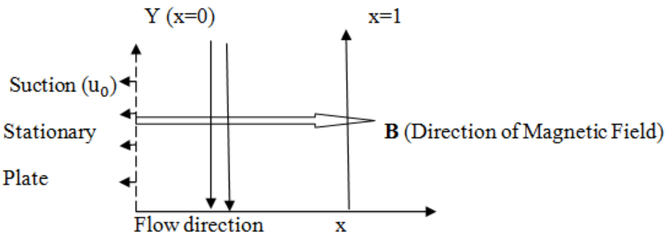

Figure 1.

IV. GOVERNING EQUATIONS

The following assumptions are made in order to derive governing equations of the flow.

1. The fluid is assumed to be incompressible with constant density and small Reynolds number.

2. The plates are electrically non-conducting and very long in the y –direction.

3. Thermal conductivity, electrical conductivity and co-efficient of viscosity are constant.

4. The fluid is Newtonian, does not undergo any chemical reaction and observes no slip conditions at the plates.

5. Effects of Hall current are ignored since a weak magnetic field is applied hence generalized Ohm’s law negligible.

Based on these assumptions and the flow being steady, the governing equations are given by;

$$\rho\left(-u_0\frac{\partial u}{\partial x}\right) = \mu\frac{\partial^2 u}{\partial x^2} - \sigma\mu_e^2 H^2 u \text{ (Navier stokes equation)} \quad (1)$$

and the energy equation expressed as

$$\rho C_p\left(\frac{\partial T}{\partial t}\right) = k\frac{\partial^2 T}{\partial x^2} + \mu\left(\frac{\partial u}{\partial x}\right)^2 + \sigma B_0^2 U^2 \text{(Energy equation)} \quad (2)$$

Where

u_0 is suction velocity, ϑ Kinematic viscosity, Cp is specific heat at constant pressure, σ is Electrical conductivity, T is temperature of the fluid, ρ is Fluid density, $-\sigma B_0^2 U$ is lorentz force, μ is Coefficient of viscosity, k is Thermal conductivity, D/(DT) is Material derivative,$\sigma B_0^2 U$ is internal heat generation

The initial and boundary conditions are; at x = 0, u = 0 and $T = T_0$ while at x = 1, u = U and $T = T_1$.

When the following non-dimensional parameters are introduced;

$P_r = \frac{C_p\mu}{k}$ -Prandtl number and $M^2 = \frac{\sigma B_{0\vartheta}^2}{\rho U^2}$- Hartmann number.

Equation (1) becomes

$$\frac{\partial^2 u}{\partial x^2} + u_0\frac{\partial u}{\partial x} - M^2 u = 0 \quad (3)$$

Similarly equation (2) reduces to

$$\frac{\partial\theta}{\partial t} = \frac{1}{P_r}.\frac{\partial^2\theta}{\partial x^2} \quad (4)$$

where θ Dimensionless fluid temperature , T_0 and T_1 are temperatures for the porous plate and impulsively started plate respectively.

For simplicity, the corresponding initial and boundary conditions in dimensional form are taken to be;

At x = 0, u = 0 and θ = 0 while at x =1, u = 1 and θ = 1 (5)

V. METHOD OF SOLUTION

The non-linear differential equations (3) and (4) together with initial and boundary conditions (5) are solved using finite difference approach. This is a linearization technique in which derivatives occurring in the generated non-linear differential equations are replaced by their central finite difference approximations. Thus equations (3) and (4) when expressed in finite difference of second order give rise to equations (6) and (7) respectively.

$$\frac{U_{ij+1}-2U_{ij}+U_{ij-1}}{k^2}+u_0\frac{U_{ij+1}-U_{ij-1}}{2k}-M^2u_{ij}=0 \quad (6)$$

$$\frac{\theta_{ij+1}-\theta_{ij-1}}{k}=\frac{1}{p_r}\frac{\theta_{ij+1}-2\theta_{ij}+\theta_{ij-1}}{k^2} \quad (7)$$

Where i and j refer to x and t respectively.

The finite difference equations (6) and (7) are then solved using MATLAB PDE software.

VI. RESULTS AND DISCUSSIONS

The flow parameters Hartmann and Prandtl Numbers are found to considerably affect velocity profiles and temperature distribution of the flow.

TABLE I. EFFECT OF DIFFERENT VALUES OF HARTMANN ON VELOCITY DISTRIBUTION

Hartman numbers	Velocity profiles at different values of y										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
U at M= 1	1.0000	0.7893	0.6106	0.4571	0.3275	0.2206	0.1354	0.0712	0.0274	0.0037	0
U at M =3	1.0000	0.7572	0.5610	0.4008	0.2719	0.1705	0.0939	0.0397	0.0065	-0.0067	0
U at M = 5	1.000	0.7306	0.5215	0.3575	0.2307	0.1348	0.0653	0.0188	-0.0070	-0.0132	0
U at M = 7	1.000	0.7077	0.4886	0.3228	0.1989	0.1083	0.0449	0.0045	-0.0159	-0.0173	0

TABLE II. EFFECT OF DIFFERENT VALUES OF PRANDTL NUMBER ON TEMPERATURE DISTRIBUTION

Prandtl numbers	Temperature at different values of y										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Pr = 0.5	1.0000	0.7630	0.5739	0.4208	0.2979	0.2006	0.1252	0.0692	0.0304	0.0076	0
Pr = 0.8	1.0000	0.7845	0.6072	0.4584	0.3342	0.2318	0.1491	0.0848	0.0383	0.0098	0
Pr = 1.0	1.000	0.8063	0.6414	0.4975	0.3726	0.2653	0.1751	0.1021	0.0472	0.012	0

Table 1 shows the effect of Hartmann number on velocity profiles. It is found that an increase in the Hartmann number leads into a decrease in velocity distribution. This is in agreement with the physical situation since Lorentz force generated due to the application of a constant magnetic field. This force being resistive, opposes the fluid motion hence decelerating the flow. In table, it is also observed that Prandtl number affects temperature distribution. This is because thermal diffusion is higher near to the impulsively started plate. Similarly, boundary layer thickness reduce as Prandtl number increase as this indicates a slow rate in thermal diffusion. The corresponding figures for velocity and temperature distribution for tables 1 and 2 are as follows:

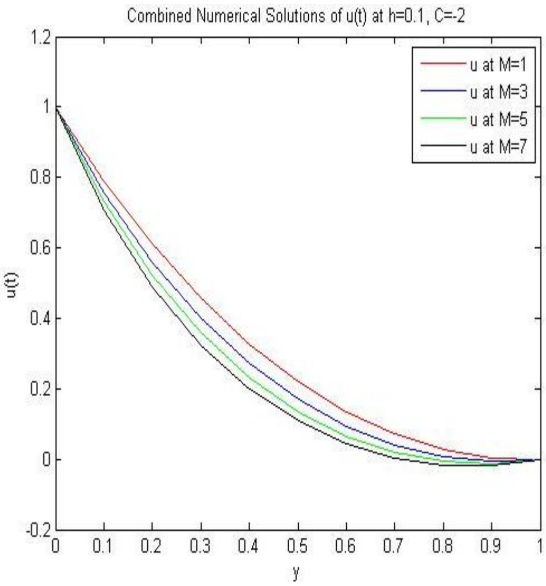

Figure 2. Velocity profiles resulting from variation of Hartmann number

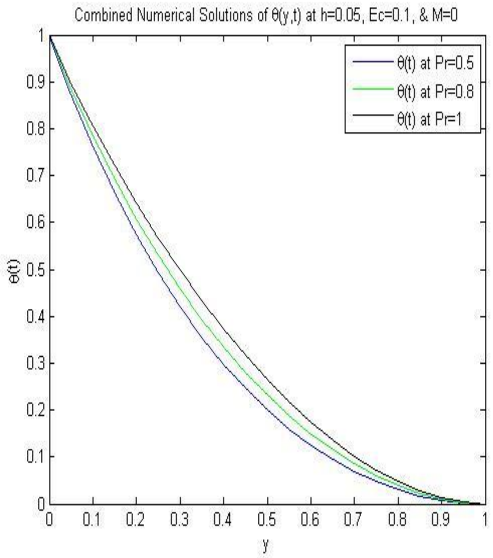

Figure 3. Temperature distribution on variation of Prandtl number

VII. CONCLUSION

Velocity profiles and temperature distribution on a steady flow of an incompressible, viscous and electrically conducting fluid in parallel vertical plates in the presence of a uniform magnetic field have been investigated. Hartmann and Prandtl Numbers are found to have a great effect on velocity profiles and temperature distribution respectively. Results obtained for various values of these flow parameters have been found to suitably agree with the physical situation of the flow. It has been noted that an increase in Hartmann number causes a decrease in velocity profiles while an increase in Prandtl Number leads to a fall in temperature distribution.

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