

Thermal Buckling of Rectangular FGM Plate with Variation Thickness Based on the Higher Order Shear Deformation

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Abstract- Equilibrium and stability equations of a thin rectangular plate with length a , width b , and thickness $h(x) = C_1x + C_2$, made of functionally graded materials under thermal loads are derived based on the higher order shear deformation theory. It is assumed that the material properties vary as a power form of thickness coordinate variable z . The derived equilibrium and buckling equations are then solved analytically for a plate with simply supported boundary conditions. One type of thermal loading, uniform temperature rise and gradient through the thickness are considered, and the buckling temperatures are derived. The influences of the plate aspect ratio, the relative thickness, the gradient index and the transverse shear on buckling temperature difference are all discussed.

Keywords- Stability of plate; Thermal buckling; Rectangular plate; Functionally graded material; higher order shear deformation theory.

I. INTRODUCTION

In recent years, functionally graded materials (FGMs) which named by a group of material scientists in Japan [1] in 1984, have attracted much interest as heat shielding materials for aircraft, space vehicles and other engineering applications. Functionally graded materials are composite materials, which are microscopically inhomogeneous, and the mechanical properties vary smoothly or continuously from one surface to the other. It is this continuous change that results in gradient properties in functionally graded materials. Typically, these materials are made from a mixture of metal and ceramic, or a combination of different metals. Unlike fiber matrix composites which have a strong mismatch of mechanical properties across the interface of two discrete materials bonded together and may result in debonding at high temperatures, functionally graded materials have the advantage of being able to survive environment with high temperature gradient, while maintaining their structural integrity.

The ceramic material provides high temperature resistance due to its low thermal conductivity, while the ductile metal component prevents fracture due to thermal stresses. Furthermore, a mixture of ceramic and metal with a

continuously varying volume fraction can be easily manufactured. In view of the advantages of functionally graded materials, a number of investigations dealing with thermal stresses had been published in the scientific literature. In recent years, Tanigawa et al. [2] derived a one dimensional temperature solution for a non-homogeneous plate in transient state and also optimized the material composition by introducing a laminated composite model. Analytical formulation and numerical solution of the thermal stresses and deformations for axisymmetrical shells of FGM subjected to thermal loading due to fluid was obtained by Takezono et al. [3]. Aboudi et al. developed a new kind of higher order shear deformation theory for functionally graded materials that explicitly couples the micro-structural and macro structural effects[4]. The response of a functionally graded ceramic-metal plate was investigated by Praveen and Reddy using a finite element model that accounts for the transverse shear strains, rotary inertia, and moderately large rotations in the Von Karman sense [5]. In Ref. [6], Reddy et al. developed the relationship between the bending solutions of the classical plate theory and the first order plate theory for functionally graded circular plates. Sumi studied the propagation and reflection of thermal and mechanical waves in FGMs under impulsive heat addition [7]. Javaheri and Eslami reported mechanical and thermal buckling of rectangular functionally graded plates (FGPs) based on the classical plate theory[8,9]. They used energy method and reached to the closed form solutions. They derived equilibrium and stability equations for functionally graded plates are identical to the equations for laminated composite plates. They have also investigated thermal buckling of FGPs based on the higher order displacement field [10]. They obtained buckling loads by solving the system of five stability equations. Motivated by Javaheri, Lanhe studied thermal buckling of moderately thick rectangular FGPs based on the first order shear deformation theory [11]. Considerable research has also been performed on the analysis of the stresses and deformations of functionally graded structures. However, Buckling analyses of FGM structures are scarce in the open literature. A formulation of the stability problem for FGM plates was presented by Birman [12] where a micro-mechanical model was employed to solve the buckling problem for a rectangular plate subjected to uniaxial compression. The stability of a functionally graded cylindrical shell under axial harmonic loading was investigated by Ng et

al. [13]. Recently, Wu et al. presented the thermal buckling analysis of a simply supported thin rectangular FGM plate based on the classical plate theories [16]. In that paper, we initially consider an FGM rectangular thin flat plate of length a , width b , and thickness $h(x) = C_1x + C_2$, subjected to the thermal loads. The material properties are assumed to vary as a power form of thickness coordinate variable, the linear stability equations are derived using the critical equilibrium method, and then the closed form of solutions for the linear stability equations is presented. They also investigated the influence of neutral plane deformation, the aspect ratio, the relative thickness, and the graded index of the plate on the critical buckling temperature difference. In view of the fact that one solution to buckling of linear variational thickness plates under thermal loads exist, an attempt is made to solve the thermal buckling problem of a functionally graded plate with moderately thickness and simply supported boundary conditions. In this paper, the stability equations are established based on the higher order shear deformation theory. Then five equations are combined into one governing equation with respect to w by eliminating the other variables. At last, the analytical solution for this equation is presented and the influence of transverse shear deformation on buckling is discussed. In our study, one kind of thermal loading, uniform temperature rise and gradient through the thickness are considered. Functionally Graded Plates (FGMs) are typically made from a mixture of ceramics and metal or a combination of different metals. The ceramic constituent of the material provides the high-temperature resistance due to its low thermal conductivity. The ductile metal constituent, on the other hand, prevents fracture caused by stresses due to high-temperature gradient in a very short period of time. Further, a mixture of a ceramic and a metal with a continuously varying volume fraction can be easily manufactured.

II. MATERIAL PROPERTIES

Consider a rectangular plate made of a mixture of metal and ceramic. The material in top surface and in bottom surface is metal and ceramic respectively. The modulus of elasticity E , the coefficient of thermal expansion α and the Poisson's ratio ν are assumed as

$$\begin{aligned} E(z) &= E_c V_c + E_m (1 - V_c), \\ E(z) &= \alpha_c V_c + \alpha_m (1 - V_c), \nu(z) = \nu_0 \end{aligned} \quad (1)$$

where E_m and α_m denote the elastic moduli and the coefficient of thermal expansion of metal respectively; E_c and α_c denote the elastic moduli and the coefficient of thermal expansion of ceramic respectively; V_c denotes the volume fraction of the ceramic and is assumed as a power function as follows:

$$V_c = \left(\frac{2z+h}{2h} \right)^k, V_m = 1 - V_c \quad (2)$$

Where z is the thickness coordinate variable; and $(-h/2 \leq z \leq h/2)$, where h is the thickness of the plate and k is the power law index that takes values greater than or equals to zero. Substituting Eq.(2) into Eq.(1), material properties of the FGM plate are determined, which are the same as the equations proposed by Praveen and Reddy [5].

$$\begin{aligned} E(z) &= E_m + E_{cm} \left(\frac{2z+h}{2h} \right)^k, \\ \alpha(z) &= \alpha_c V_c + \alpha_{cm} \left(\frac{2z+h}{2h} \right)^k, \nu(z) = \nu_0 \end{aligned} \quad (3)$$

Where

$$E_{cm} = E_c - E_m, \alpha_{cm} = \alpha_c - \alpha_m \quad (4)$$

III. EQUILIBRIUM AND STABILITY EQUATIONS

We initially consider an FGM rectangular thin plate of length a , width b , and thickness $h(x) = C_1x + C_2$, subjected to the thermal loads. Rectangular Cartesian coordinates (x, y, z) are assumed for derivations. According to the higher order shear deformation theory, the strains of the plate can be expressed as:

$$\begin{aligned} \varepsilon_x &= u_{,x} + \frac{1}{2} w_{,x}^2 & \varepsilon_y &= v_{,y} + \frac{1}{2} w_{,y}^2 \\ \varepsilon_{xy} &= u_{,y} + v_{,x} + w_{,x} w_{,y} & \varepsilon_{xz} &= u_{,z} + w_{,x} & \varepsilon_{yz} &= v_{,z} + w_{,y} \end{aligned} \quad (5)$$

where ε_x and ε_y are the normal strains and $\varepsilon_{xy}, \varepsilon_{xz},$ and ε_{yz} are the shear strains. Here u, v and w denote the displacement components in the x, y and z directions, respectively, and a comma indicates the partial derivative. According to the first order shear deformation theory, used in the present study is based on the following displacement:

$$\begin{aligned} u(x, y) &= u_0(x, y) + z u_1(x, y) + z^2 u_2(x, y) + z^3 u_3(x, y) \\ v(x, y) &= v_0(x, y) + z v_1(x, y) + z^2 v_2(x, y) + z^3 v_3(x, y) \\ w(x, y) &= w_0(x, y) \end{aligned} \quad (6)$$

Where

$$\varepsilon_{xy} = \varepsilon_{yz} = 0 \quad \text{at} \quad z = \pm \frac{h}{2} \quad (7)$$

and

$$u_2 = v_2 = 0, u_3 = -\frac{4z^3}{3h^2}(u_1 + w_{0,x}), v_3 = -\frac{4z^3}{3h^2}(v_1 + w_{0,y}) \quad (8)$$

Substituting Eqs. (6) into nonlinear strain-displacement relations (5) gives the kinematic relations as

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{pmatrix} = \begin{pmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \end{pmatrix} + z \begin{pmatrix} k_x^0 \\ k_y^0 \\ k_{xy}^0 \end{pmatrix} + z^3 \begin{pmatrix} k_x^2 \\ k_y^2 \\ k_{xy}^2 \end{pmatrix}, \begin{pmatrix} \varepsilon_{xz} \\ \varepsilon_{yz} \end{pmatrix} = \begin{pmatrix} \varepsilon_{xz}^0 \\ \varepsilon_{yz}^0 \end{pmatrix} + z^2 \begin{pmatrix} k_{xz}^1 \\ k_{yz}^1 \end{pmatrix} \quad (9)$$

where

$$\begin{pmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \end{pmatrix} = \begin{pmatrix} u_{0,x} + \frac{1}{2} w_{0,x}^2 \\ v_{0,y} + \frac{1}{2} w_{0,y}^2 \\ u_{0,y} + v_{0,x} + w_{0,x} w_{0,y} \end{pmatrix}, \begin{pmatrix} \varepsilon_{xz}^0 \\ \varepsilon_{yz}^0 \end{pmatrix} = \begin{pmatrix} u_1 + w_{0,x} \\ u_1 + w_{0,y} \end{pmatrix}$$

$$\begin{pmatrix} k_x^0 \\ k_y^0 \\ k_{xy}^0 \end{pmatrix} = \begin{pmatrix} u_{1,x} \\ v_{1,y} \\ u_{0,y} + v_{0,x} \end{pmatrix}, \begin{pmatrix} k_{xz}^1 \\ k_{yz}^1 \end{pmatrix} = \begin{pmatrix} -\frac{4}{3h^2}(u_1 + w_{0,x}) \\ -\frac{4}{3h^2}(u_1 + w_{0,y}) \end{pmatrix}$$

$$\begin{pmatrix} k_x^2 \\ k_y^2 \\ k_{xy}^2 \end{pmatrix} = \begin{pmatrix} -\frac{4}{3h^2}(u_{1,x} + w_{0,xx}) \\ -\frac{4}{3h^2}(v_{1,y} + w_{0,yy}) \\ \frac{4}{3h^2}(u_{1,y} + v_{1,x} + 2w_{0,xy}) \end{pmatrix} \quad (10)$$

Hooke's law for a plate is defined as:

$$\begin{aligned} \sigma_x &= \frac{E(z)}{1-\nu_0^2} [\varepsilon_x + \nu_0 \varepsilon_y - (1+\nu_0)\alpha(z)T] \\ \sigma_y &= \frac{E(z)}{1-\nu_0^2} [\varepsilon_y + \nu_0 \varepsilon_x - (1+\nu_0)\alpha(z)T] \\ \sigma_{xy} &= \frac{E(z)}{2(1+\nu_0)} \varepsilon_{xy}, \sigma_{xz} = \frac{E(z)}{2(1+\nu_0)} \varepsilon_{xz}, \sigma_{yz} = \frac{E(z)}{2(1+\nu_0)} \varepsilon_{yz} \end{aligned} \quad (11)$$

The forces and moments per unit length of the plate expressed in terms of the stress components through the thickness are

$$(N_i, M_i) = \int_{-h/2}^{h/2} \sigma_i(1, z) dz \quad i = x, y, xy \quad (12)$$

Substituting Eqs. (3), (9), and (11) into Eqs. (12), gives the constitutive relations as

$$\begin{aligned} (N_x, M_x) &= \frac{1}{1-\nu_0^2} \left[(E_1, E_2)(\varepsilon_x^0 + \nu_0 \varepsilon_y^0) + (E_1, E_2)(k_x^0 + \nu_0 k_y^0) \right. \\ &\quad \left. + (E_4, E_5)(k_x^2 + \nu_0 k_y^2) - (1+\nu_0)(\phi_1, \phi_2) \right] \\ (N_y, M_y) &= \frac{1}{1-\nu_0^2} \left[(E_1, E_2)(\varepsilon_y^0 + \nu_0 \varepsilon_x^0) + (E_2, E_3)(k_y^0 + \nu_0 k_x^0) \right. \\ &\quad \left. + (E_4, E_5)(k_y^2 + \nu_0 k_x^2) - (1+\nu_0)(\phi_1, \phi_2) \right] \\ (N_{xy}, M_{xy}) &= \frac{1}{2(1+\nu_0)} \left[(E_1, E_2)\varepsilon_{xy}^0 + (E_2, E_3)k_{xy}^0 + (E_4, E_5)k_{xy}^2 \right] \end{aligned} \quad (13)$$

Where

$$\begin{aligned} (E_1, E_2, E_3, E_4, E_5, E_6, E_7) &= \int_{-h/2}^{h/2} (1, z, z^2, z^3, z^4, z^5, z^6) E(z) dz \\ (\phi_1, \phi_2, \phi_3, \phi_4) &= \int_{-h/2}^{h/2} (1, z, z^2, z^3) \alpha(z) T(x, y, z) E(z) dz \\ \psi &= \int_{-h/2}^{h/2} E(z) \alpha^2(z) T^2(x, y, z) dz \end{aligned} \quad (14)$$

$$\mu^* = \Delta T \left[E_m \alpha_m + \frac{1}{k+1} (E_m \alpha_{cm} + E_{cm} \alpha_m) + \frac{1}{2k+1} E_{cm} \alpha_{cm} \right] \quad (15)$$

The nonlinear equations of equilibrium according to Von Karman's theory are given by

$$\begin{aligned} N_{x,x} + N_{xy,y} &= 0, N_{yx,x} + N_{y,y} = 0, M_{x,x} + M_{xy,y} = 0 \\ M_{xy,x} + M_{y,y} &= 0, N_x w_{0,xx} + 2N_{xy} w_{0,xy} + N_y w_{0,yy} = 0 \end{aligned} \quad (16)$$

The stability equations of the plate may be derived by the adjacent equilibrium criterion [17]. Assume that the equilibrium state of a FGP under mechanical or thermal loads is defined in terms of the displacement components u_0, v_0 and w_0 . The displacement components of a neighboring stable state differ by u_1, v_1 and w_1 with respect to the equilibrium position. Thus, the total displacements of a neighboring state are

$$\begin{aligned} u &= u_0^0 + u_0^1, v = v_0^0 + v_0^1, w = w_0^0 + w_0^1, u_1 = u_1^0 + u_1^1 \\ v_1 &= v_1^0 + v_1^1 \end{aligned} \quad (17)$$

Similarly, the force and momentum resultants of a neighboring state may be related to the state of equilibrium as

$$N_i = N_{i0} + N_{i1}, M_i = M_{i0} + M_{i1} \quad i = x, y, xy \quad (18)$$

Where N_{x1}, N_{y1} and N_{xy1} represent the linear parts of the force increments corresponding to u_1, v_1 and w_1 . The stability equations may be obtained by substituting Eqs. (17) and (18) in Eq. (16). Upon substitution, the terms in the resulting equations with superscript 0 satisfy the equilibrium condition and therefore drop out of the equations. Also, the nonlinear terms with superscript 1 are ignored because they are small compared to the linear terms. The remaining terms form the stability equations as

$$\begin{aligned} N_{x1,x} + N_{xy1,y} &= 0, N_{yx1,x} + N_{y1,y} = 0, M_{x1,x} + M_{xy1,y} = 0 \\ M_{xy1,x} + M_{y1,y} &= 0, N_{x0} w_{0,xx}^1 + 2N_{xy0} w_{0,xy}^1 + N_{y0} w_{0,yy}^1 = 0 \end{aligned} \quad (19)$$

The superscript 1 refers to the state of stability and the superscript 0 refers to the state of equilibrium conditions.

IV. BUCKLING OF FUNCTIONALLY GRADED PLATES UNDER UNIFORM TEMPERATURE RISE

The initial uniform temperature of the plate is assumed to be T_1 . The plate is simply supported along the edges in bending and rigidly fixed in extension. Under these boundary conditions, the temperature can be uniformly raised to a final value T_2 such that the plate buckles [9]. To find the critical buckling temperature difference, $\Delta T = T_2 - T_1$ the pre-buckling thermal stresses should be found. Solving the membrane form of equilibrium equations, using the method developed by Meyers [18] in conjunction with Galerkin's formulation, gives the pre-buckling force resultants

$$\begin{aligned} N_{x0} &= -\frac{\mu^*}{2(1-\nu_0)} (C_1 a - 2C_2) \\ N_{x0} &= -\frac{\mu^*}{2(1-\nu_0)} (C_1 a - 2C_2) - \mu^* (C_1 x) \\ N_{xy0} &= 0 \end{aligned} \quad (20)$$

The simply supported boundary condition is defined as

$$\begin{aligned}
w_0^1(x, 0) &= w_0^1(x, b) = w_0^1(0, y) = w_0^1(a, y) = 0 \\
u_0^1(x, 0) &= u_0^1(x, b) = v_0^1(0, y) = v_0^1(a, y) = 0 \\
u_1^1(x, 0) &= u_1^1(x, b) = v_1^1(0, y) = v_1^1(a, y) = 0 \\
M_y(x, 0) &= M_y(x, b) = M_x(0, y) = M_x(a, y) = 0
\end{aligned} \tag{21}$$

The following approximate solution is seen to satisfy both the differential equation and the boundary conditions

$$\begin{aligned}
u_0^1(x, y) &= \sum_{m=1}^m \sum_{n=1}^n u_{0mn} \cos \alpha x \sin \beta y \\
u_1^1(x, y) &= \sum_{m=1}^m \sum_{n=1}^n u_{1mn} \cos \alpha x \sin \beta y \\
v_0^1(x, y) &= \sum_{m=1}^m \sum_{n=1}^n v_{0mn} \sin \alpha x \cos \beta y \\
v_1^1(x, y) &= \sum_{m=1}^m \sum_{n=1}^n v_{1mn} \sin \alpha x \cos \beta y \\
w_0^1(x, y) &= \sum_{m=1}^m \sum_{n=1}^n w_{0mn} \sin \alpha x \sin \beta y
\end{aligned} \tag{22}$$

Where

$$\alpha = \frac{m\pi}{a}, \quad \beta = \frac{n\pi}{b}, \quad m, n = 1, 2, 3, \dots$$

Where m and n are number of half waves in x and y directions, respectively, and $(u_{0mn}, u_{1mn}, v_{0mn}, v_{1mn}, w_{0mn})$ are constant coefficients. Substituting Eqs. (22) into the stability equations (19) and using the kinematic and constitutive relations yield a system of five homogeneous equations for $u_{0mn}, u_{1mn}, v_{0mn}, v_{1mn}$, and w_{0mn} , that is,

$$\left[k_{ij} \right] \begin{pmatrix} u_{0,mn} \\ v_{0,mn} \\ w_{0,mn} \\ u_{1,mn} \\ v_{1,mn} \end{pmatrix} = 0 \tag{23}$$

In which k_{ij} is a symmetric matrix with the components

$$\begin{aligned}
k_{11} &= E_1 \left(\frac{m^2 \pi^2}{a^2} \right) + \frac{E_1 (1 - \nu_0)}{2} \left(\frac{n^2 \pi^2}{b^2} \right) \\
k_{12} &= \frac{E_1 (1 + \nu_0)}{2} \left(\frac{m\pi}{a} \right) \left(\frac{n\pi}{b} \right) \\
k_{13} &= 0 \\
k_{14} &= \left(E_2 - \frac{4E_4}{3h^2} \right) \left[\left(\frac{m^2 \pi^2}{a^2} \right) + \frac{(1 - \nu_0^2)}{2} \left(\frac{n^2 \pi^2}{b^2} \right) \right] \\
k_{15} &= \left(\frac{E_2}{2} - \frac{2E_4}{3h^2} \right) (1 + \nu_0) \left(\frac{m\pi}{a} \right) \left(\frac{n\pi}{b} \right) \\
k_{21} &= -k_{12} \\
k_{22} &= E_1 \left[\left(\frac{n^2 \pi^2}{b^2} \right) + \frac{(1 - \nu_0)}{2} \left(\frac{m^2 \pi^2}{a^2} \right) \right]
\end{aligned}$$

$$\begin{aligned}
k_{23} &= -\frac{4E_4}{3h^2} \left[\left(\frac{m^2 \pi^2}{a^2} \right) \left(\frac{n\pi}{b} \right) + \left(\frac{n^3 \pi^3}{b^3} \right) \right] \\
k_{24} &= \left(\frac{E_2}{2} - \frac{2E_4}{3h^2} \right) (1 + \nu_0) \left(\frac{m\pi}{a} \right) \left(\frac{n\pi}{b} \right) \\
k_{25} &= \left(2 - \frac{4E_4}{3h^2} \right) \left[\left(\frac{n^2 \pi^2}{b^2} \right) + \frac{(1 - \nu_0)}{2} \left(\frac{m^2 \pi^2}{a^2} \right) \right] \\
k_{31} &= k_{13}, k_{32} = k_{23} \\
\bar{k}_{33} &= \frac{16E_7}{9h^4} \left[\left(\frac{m^2 \pi^2}{a^2} \right) + \left(\frac{n^2 \pi^2}{b^2} \right) \right] - \left(\frac{4E_2}{h^2} - \frac{E_1}{2} - \frac{8E_5}{h^4} \right) \\
&\quad (1 - \nu_0) \left[\left(\frac{m^2 \pi^2}{a^2} \right) + \left(\frac{n^2 \pi^2}{b^2} \right) \right] (1 - \nu_0^2) \left(N_{x0} \left(\frac{m^2 \pi^2}{a^2} \right) + N_{y0} \left(\frac{n^2 \pi^2}{b^2} \right) \right)
\end{aligned}$$

$$\begin{aligned}
k_{34} &= -\left(\frac{4E_3}{h^2} - \frac{E_1}{2} - \frac{8E_5}{h^4} \right) (1 - \nu_0) \left(\frac{m\pi}{a} \right) + \left(\frac{16E_7}{9h^2} - \frac{4E_5}{3h^2} \right) \times \\
&\quad \left[\left(\frac{m\pi}{a} \right)^2 + \left(\frac{m\pi}{a} \right) \left(\frac{n\pi}{b} \right) \right] \\
k_{35} &= -\left(\frac{4E_3}{h^2} - \frac{E_1}{2} - \frac{8E_5}{h^4} \right) (1 - \nu_0) \left(\frac{m\pi}{a} \right) + \left(\frac{16E_7}{9h^2} - \frac{4E_5}{3h^2} \right) \times \\
&\quad \left[\left(\frac{m\pi}{a} \right)^2 + \left(\frac{m\pi}{a} \right) \left(\frac{n\pi}{b} \right) \right] \\
k_{41} &= -k_{14}, k_{42} = -k_{24}, k_{43} = -k_{34}
\end{aligned}$$

$$k_{44} = \left(\frac{8E_5}{3h^2} - \frac{16E_7}{9h^4} - E_3 \right) \left[\left(\frac{m^2 \pi^2}{a^2} \right) + \frac{(1 - \nu_0)}{2} \left(\frac{n^2 \pi^2}{b^2} \right) \right] -$$

$$\left(\frac{E_1}{2} - \frac{4E_3}{h^2} + \frac{8E_5}{h^4} \right) (1 - \nu_0)$$

$$k_{45} = \left(\frac{4E_5}{3h^2} - \frac{E_3}{2} - \frac{9E_7}{9h^4} - E_3 \right) \left(\frac{m\pi}{a} \right) \left(\frac{n\pi}{b} \right)$$

$$k_{51} = -k_{15}, k_{52} = -k_{25}, k_{53} = -k_{35}, k_{54} = k_{45}$$

$$k_{55} = -\left(\frac{8E_5}{3h^2} - \frac{16E_7}{9h^4} - E_3 \right) \left[\left(\frac{n^2 \pi^2}{b^2} \right) + \frac{(1 - \nu_0)}{2} \left(\frac{m^2 \pi^2}{a^2} \right) \right]$$

$$+ \left(\frac{E_1}{2} - \frac{4E_3}{h^2} + \frac{8E_5}{h^4} \right) (1 - \nu_0)$$

$$\bar{k}_{33} = k_{33} - \mu^* \left[\frac{\pi^2 (1 + \nu_0)}{2b^2} (C_1 a + 2C_2) \left(\left(\frac{m^2 b^2}{a^2} + n^2 \right) \right) \right]$$

$$\bar{k}_{33} = k_{33} - \Delta T \left[E_m \alpha_m + \frac{(E_m \alpha_{cm} + E_{cm} \alpha_m)}{k + 1} + \frac{E_{cm} \alpha_{cm}}{2k + 1} \right]$$

$$\times \left[\frac{\pi^2 (1 + \nu_0)}{2b^2} (C_1 a + 2C_2) \left(\left(\frac{m^2 b^2}{a^2} + n^2 \right) \right) \right] \tag{24}$$

Substituting pre-buckling forces from Eqs.(20) into the relation of k_{33} and setting $|k_{ij}| = 0$ to obtain the nonzero solution, the value of ΔT_{cr} is found as

$$\begin{bmatrix} k_{11} & k_{12} & & k_{13} & & k_{14} & k_{15} \\ k_{21} & k_{22} & & k_{23} & & k_{24} & k_{25} \\ & & k_{33} - \Delta T \left[E_m \alpha_m + \frac{(E_m \alpha_{cm} + E_{cm} \alpha_m)}{k+1} + \frac{E_{cm} \alpha_{cm}}{2k+1} \right] & & & k_{34} & k_{35} \\ k_{31} & k_{32} & & \left[\frac{\pi^2(1+\nu_0)}{2b^2} (C_1 a + 2C_2) \mu^s \left(\frac{m^2 b^2}{a^2} + n^2 \right) \right] & & & \\ & & & & & & \\ k_{41} & k_{42} & & k_{43} & & k_{44} & k_{45} \\ k_{51} & k_{52} & & k_{53} & & k_{54} & k_{55} \end{bmatrix} = 0 \quad (25)$$

$$\Delta T_{cr} = \frac{b^2 K_d}{\pi^2(1+\nu_0)(C_1 a + 2C_2) \mu^s \left[\frac{m^2 b^2}{a^2} + n^2 \right] K_c} \quad (26)$$

Where

$$K_d = \det |k_{ij}| \quad i, j = 1, 2, 3 \quad (27)$$

and

$$\begin{aligned} K_c = & k_{15}k_{24}k_{42}k_{52} + k_{12}k_{25}k_{44}k_{51} + k_{14}k_{22}k_{45}k_{51} \\ & + k_{14}k_{25}k_{41}k_{52} + k_{15}k_{21}k_{44}k_{52} + k_{11}k_{24}k_{45}k_{52} \\ & + k_{15}k_{22}k_{41}k_{54} + k_{11}k_{25}k_{42}k_{54} + k_{12}k_{21}k_{45}k_{54} \\ & + k_{12}k_{24}k_{41}k_{55} + k_{14}k_{21}k_{42}k_{55} + k_{11}k_{22}k_{44}k_{55} \\ & - k_{14}k_{25}k_{42}k_{51} - k_{15}k_{22}k_{44}k_{51} - k_{12}k_{24}k_{45}k_{51} \\ & - k_{15}k_{24}k_{41}k_{52} - k_{11}k_{25}k_{44}k_{52} - k_{14}k_{21}k_{45}k_{52} \\ & - k_{12}k_{25}k_{41}k_{54} - k_{15}k_{21}k_{42}k_{54} - k_{11}k_{22}k_{45}k_{54} \\ & - k_{14}k_{22}k_{41}k_{54} - k_{11}k_{24}k_{42}k_{55} - k_{12}k_{21}k_{44}k_{55} \end{aligned} \quad (28)$$

The critical temperature difference ΔT_{cr} is obtained for the values of m and n that make the preceding expression a minimum. By setting the power law index equal to one ($k=1$), Eq. (26) is reduced to the critical temperature difference for an FGP with a linear composition of ceramics and metal. In addition, by setting the power law index equal to zero ($k=0$) Eq. (26) is reduced to the critical temperature difference of homogeneous plates.

V. ILLUSTRATION

To illustrate the proposed approach, a ceramic-metal FGP is considered.

Variation of the critical temperature difference ΔT_{cr} versus the aspect ratio b/a , and power law index k are listed for one loading cases in Tables 1. In each table, the values of the critical temperature difference ΔT_{cr} obtained by the method developed in the present article based on higher order theory are compared with respective values obtained based on classical plate theory [9]. In Tables 1,2 the results of buckling analysis for the plate under uniform temperature rise are

presented. Figure 1,2 show that the buckling temperature increases by the increase of the aspect ratio b/a and decreases with increase of the power law index k from 0 to 10. The combination of materials consists of aluminum and alumina. The Young's modulus, coefficient of thermal expansion and thermal conductivity, which has come in the following table. Poisson's ratio is chosen to be 0.3. The plate is assumed to be simply supported on all four edges.

constituent materials	
aluminum	alumina
$E_m = 70Gpa$	$E_c = 380Gpa$
$\alpha_m = 23 \times \frac{10^{-5}}{c^0}$	$\alpha_c = 7.4 \times \frac{10^{-5}}{c^0}$
$k_m = 204W/mk^5$	$k_c = 204W/mk^5$

TABLE I. CRITICAL BUCKLING TEMPERATURE OF THE FGP UNDER UNIFORM TEMPERATURE RISE DUE TO HIGHER ORDER (H) AND FIRST ORDER (F) THEORIES WITH RESPECT TO k AND b/a ($C_1=C_2=.005$)

k		$b/a=1$	$b/a=2$	$b/a=3$	$b/a=4$	$b/a=5$
0	H	17.011	27.398	85.212	145.152	222.167
	F	17.088	26.88	85.252	144.64	220.667
1	H	7.645	9.533	39.574	67.376	103.004
	F	7.939	9.835	39.624	67.25	102.634
5	H	7.087	7.983	36.108	61.435	94.134
	F	7.26	8.132	36.203	61.395	93.605
10	H	7.218	8.401	37.137	63.149	96.879
	F	7.462	8.636	37.2	63.068	96.12

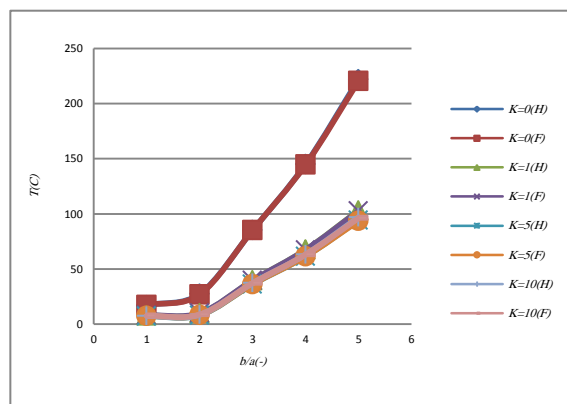


Figure 1. Critical buckling temperature of the FGP under uniform temperature rise due to higher order (H) and first order (F) theories with respect to k and b/a ($C_1=C_2=.005$)

TABLE II. CRITICAL BUCKLING TEMPERATURE OF THE FGP UNDER UNIFORM TEMPERATURE RISE DUE TO HIGHER ORDER (H) AND FIRST ORDER

(F) THEORIES WITH RESPECT TO k AND $b/a = 1, (C_1, C_2 = .05)$

k		$C_1=1 \times 10^{-2}$	$C_1=2 \times 10^{-2}$	$C_1=3 \times 10^{-2}$	$C_1=4 \times 10^{-2}$	$C_1=5 \times 10^{-2}$
0	H	26.912	31.288	36.012	41.074	46.753
	F	27.012	31.388	36.082	41.124	46.853
1	H	6.667	8.209	10.048	12.687	15.006
	F	6.709	8.265	10.108	12.725	15.036
5	H	6.212	7.477	9.312	11.523	13.918
	F	6.296	7.507	9.365	11.603	14.018
10	H	5.292	6.897	8.698	10.501	12.868
	F	5.382	7.025	8.765	10.603	13.018

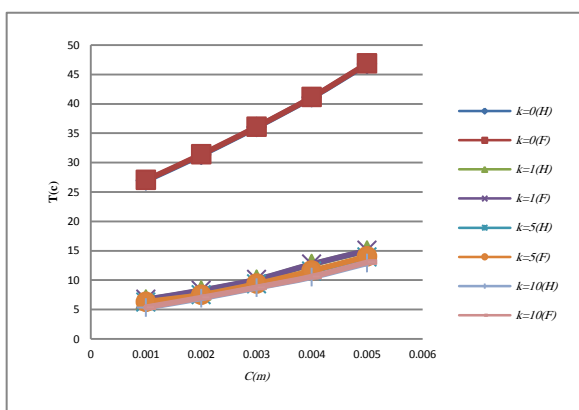


Figure 2. Critical buckling temperature of the FGP under uniform temperature rise due to higher order (H) and first order (F) theories with respect to k and $b/a = 1, (C_1, C_2 = .05)$

It is interesting to note that the buckling temperatures for homogeneous plates ($k = 0$) are considerably higher than those for the FGPs ($k > 0$) especially for the comparatively longer and thicker plates. The critical buckling temperatures obtained based on first order plate theory are noticeably greater than values obtained based on higher order shear deformation theory. The differences are considerable for long and thin plates.

VI. CONCLUSION

In this research article, equilibrium and stability equations for rectangular simply supported FGPs are obtained. The derivation is based on the higher order shear deformation theory, with the assumption of power law composition for the constituent materials. The buckling analysis of FGPs under one types of thermal loading are presented. Closed form solutions for the critical buckling temperatures of plates are presented. It is concluded that the equilibrium and stability equations are identical to the corresponding equations for laminated composite plates. The critical buckling temperature differences

ΔT_{cr} for the FGPs are generally lower than the corresponding values for homogeneous plates. Functionally graded plates have many of the same advantages as heat resistant material, but it is important to check their strength due to the thermal buckling. The critical buckling temperature difference ΔT_{cr} for FGPs is increased by increasing the aspect ratio b/a . The higher order shear deformation theory underestimates the buckling load compared with the classical plate theory.

1. The equilibrium and stability equations are identical to the corresponding equations for laminated composite plates.
2. The critical buckling temperature differences ΔT_{cr} for the FGPs are generally lower than the corresponding values for homogeneous plates. Functionally graded plates have many of the same advantages as heat resistant material, but it is important to check their strength due to the thermal buckling.
3. The critical buckling temperature difference ΔT_{cr} for FGPs is increased by increasing the aspect ratio b/a .
4. The critical buckling temperature difference ΔT_{cr} for FGPs is decrease by increasing the aspect ratio k .
5. For different b/a ratio decrease The critical buckling temperature difference ΔT_{cr} for FGPs decreases by increasing the aspect ratio k .
6. For different b/h ratio decrease The critical buckling temperature difference ΔT_{cr} for FGPs decreases by increasing the aspect ratio k .
7. For constant b/a and increase C_1 ratio decrease The critical buckling temperature difference ΔT_{cr} for FGPs decreases by increasing the aspect ratio k .
8. The critical buckling temperature difference ΔT_{cr} for FGPs is increase by increasing index C_1 .

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