International Journal of
Science and Engineering Investigations vol. 4, issue 38, March 2015
ISSN: 2251-8843

# Computer Simulation of the Cutting by Generation of Polygonal Profiles

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***Abstract***- The paper proposes a continuous method of processing the polygonal profiles, which allows processing of profiles with even and odd number of sides; this method of manufacturing of polygonal profiles implies cutting by generation (rolling). This method is based on the analogy with rack and pinion gear, and consists of the gearing motion between the semi-product and a cog rack cutter. The generation kinematics is similar to the processing of the helical-toothed wheels with planning tool. The paper aims to determine the generalized relations of the geometry of cog rack cutter geometry. Processing simulations, of the same polygonal profile, are proposed, using different geometry of the cog rack cutter; simulations of processing different polygonal profiles (with even and odd number of sides) with the same tool are also presented.

***Keywords***- *cog rack cutter, computer simulation, cutting by generation, polygonal profile*

## I. INTRODUCTION

The processing of polygonal surfaces can be accomplished by conventional machining processes, for example, using side by side milling and division with a dividing head, or by means continuous processing procedures. Among the latter we mention machining process using automatic lathes with turning heads, by which line segments are approximated with cycloid arcs [1], [2].

However by continuously processing of the surfaces, only the polygonal surfaces with an even number of sides can be processed and the deviations from the theoretical form depend on the degree of approximation achievable by the cycloid curves [3].

The processing solution using cutting by generation of polygonal surfaces, proposed by this paper, ensures high productivity, high precision of machining and a high quality of surfaces (analogy to slotting process) and also allows processing of polygonal profiles with odd number of sides.

This method is based on the imposing of gearing motion between the semi-product and tool, by means of a corresponding feeding mechanism. Polygonal profile will result as a hull of the successive positions of profile tool (such the cog rack cutter) [3] [4].

From a theoretical perspective, the simulation process involves the following steps:

- the mathematical modeling of tool profile;
- the mathematical modeling of the generating motions;
- viewing of generation process.

## II. ASPECTS REGARDING THE MODELING AND THE DESIGN OF THE COG RACK CUTTER

The profile modeling of cog rack cutter for processing polygonal profiles will be achieved based on analogy with the gearing: wheel (semi-product) - cog rack (cutter); in this modeling the following notations are used (see Fig. 1, Fig. 2, Fig. 3) [3] [4]:

- $r_d$, $d_d$: the radius and respectively the diameter of reference circle;
- $r_r$, $d_r$: the radius and respectively the diameter of rolling circle;
- $x_c$, $y_c$: coordinates of the arc centre (C);
- $x_c'$, $y_c'$: the derivatives of the coordinates $x_c$, $y_c$ in relation to angle of rotation “a”;
- $\delta_s$: the displacement of tool.

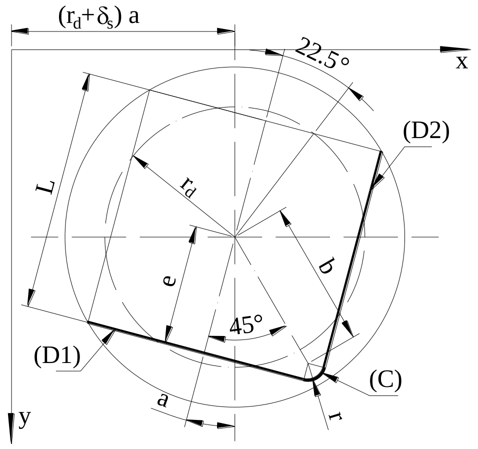
Considering both the calculation schemes proposed for square profile (Fig. 1), triangular profile (Fig. 2) and hexagonal profile (Fig. 3), and relations characteristic of these profiles, the following general relations result:

$$e = L/\left(2tg(\pi/z)\right), \tag{1}$$

$$r_d = e/cos\left(\pi/(2z)\right), \tag{2}$$

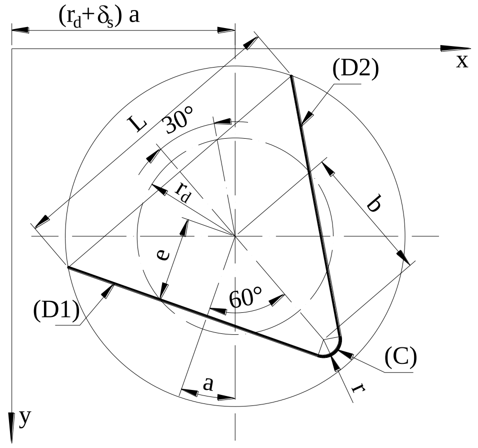
$$b = (e - r)/cos(\pi/z), \tag{3}$$

$$\begin{cases} x_c = a(r_d + \delta_s) + (e-r)sin(\pi/z - a)/cos(\pi/z), \\ y_c = (e-r)cos(\pi/z - a)/cos(\pi/z), \end{cases} \tag{4}$$



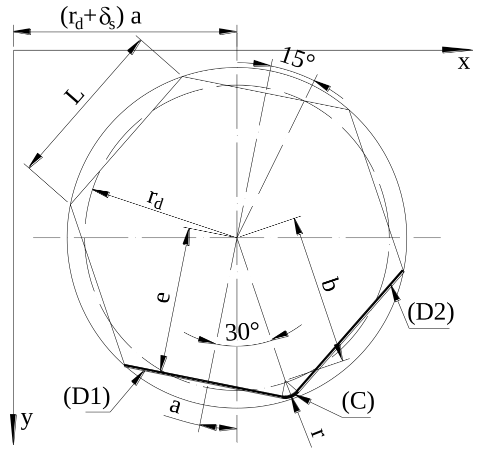

$$e = L/(2tg(\pi/4)),\ r_d = e/cos(\pi/8),$$
$$b = (e - r)/cos(\pi/4)$$

Figure 1. Square profile




$$e = L/(2tg(\pi/3)),\ r_d = e/cos(\pi/6),$$
$$b = (e - r)/cos(\pi/3)$$

Figure 2. Triangular profile




$$e = L/(2tg(\pi/6)),\ r_d = e/cos(\pi/12),$$
$$b = (e - r)/cos(\pi/6)$$

Figure 3. Hexagonal profile

$$\begin{cases} x_c' = (r_d + \delta_s) - (e - r)cos(\pi/z - a)/cos(\pi/z), \\ y_c' = (e - r)sin(\pi/z - a)/cos(\pi/z). \end{cases} \quad (5)$$

The equations of the straight line hull (D1) are:

$$\begin{cases} x = a(r_d + \delta_s) - e\,sin(a) + (r_d + \delta_s)sin(a)cos(a), \\ y = e\,cos(a) + (r_d + \delta_s)sin^2(a). \end{cases} \quad (6)$$

The equations of straight line hull (D2) are:

$$\beta = 2\pi/z - a,$$

$$\begin{cases} x = a(r_d + \delta_s) + e\,sin(\beta) - (r_d + \delta_s)sin(\beta)cos(\beta), \\ y = e\,cos(\beta) + (r_d + \delta_s)sin^2(\beta), \end{cases} \quad (7)$$

The equations of connection arc hull (C) are:

$$\begin{cases} x = x_c + r\,y_c' / \sqrt{(x_c')^2 + (y_c')^2}, \\ y = y_c + \sqrt{r^2 + (x - x_c)^2}. \end{cases} \quad (8)$$

The paper aims to model the profile of cog rack cutter for the processing of polygonal profiles with even or odd numbers of sides.

It starts from the fact that a polygonal profile is formed by segments straight line (D1) and (D2) forming between them an angle of $\pi$ - $2\pi/z$. As a result, the hulls of families formed by the successive positions of the segments (D1), (D2) and by the connection arc between these (C) will be determined (equations (6), (7), (8)) [4].

By determining the points of contact between these hulls, the equations of the tool profile can be determined completely. The determination of these points of contact is carried out by solving the system of equations formed from the equations of the hulls of segments (D1), (D2) and respectively of arc (C), relations (6), (7) and (8). Thus, the following expressions of the angles $a_1$ and $a_2$ are obtained (these determine the contact points):

$$((r_d + \delta_s)sin(a) - (e - r)tg(\pi/z))^2 = 0, \quad (9)$$

$$sin(a) = (e - r)tg(\pi/z)/(r_d + \delta_s), \quad (10)$$

$$a_1 = arcsin((e - r)tg(\pi/z)/(r_d + \delta_s)), \quad (11)$$

$$a_2 = 2\pi/z - a_1. \quad (12)$$

The equations of tool profile will be determined with equations (6) for values of the angle of rotation $a \le a_1$, with equations (8) for $a_1 < a < a_2$, respectively the equations (7) for

$a_2 \le a \le \pi - 2\pi / z$.

From the expressions of the tool profile, it can be seen that, these depend on the radius of the circle on which the workpiece is rolling [4].

Replacing the expression of radius of the rolling circle, $r_r = r_d + \delta_s$, and considering the conditions for the existence of angle $a_1$ (or $a_2$):

$$(e-r)tg(\pi / z)/(r_d + \delta_s) \le 1, \quad (13)$$

$$(e-r)tg(\pi / z)/(r_d + \delta_s) \ge -1. \quad (14)$$

the minimum value of the profile displacement is obtained $\delta_s$:

$$\delta_{s\,min} = -r_d - (e-r)tg(\pi / z). \quad (15)$$

By rolling the same workpiece, on circles of different diameters, profiles of different tools are obtained [4].

## III. The Processing Simulation of Polygonal Profiles

The modeling of polygonal profiles processing has involved both the formulation of general analytical relations that define the generating tool geometry and mathematical modeling of generating movements.

In this respect, based on performed modeling, a program was designed to allow the graphical visualization of polygonal profiles processing. The conceived program interface is shown in Fig. 4.

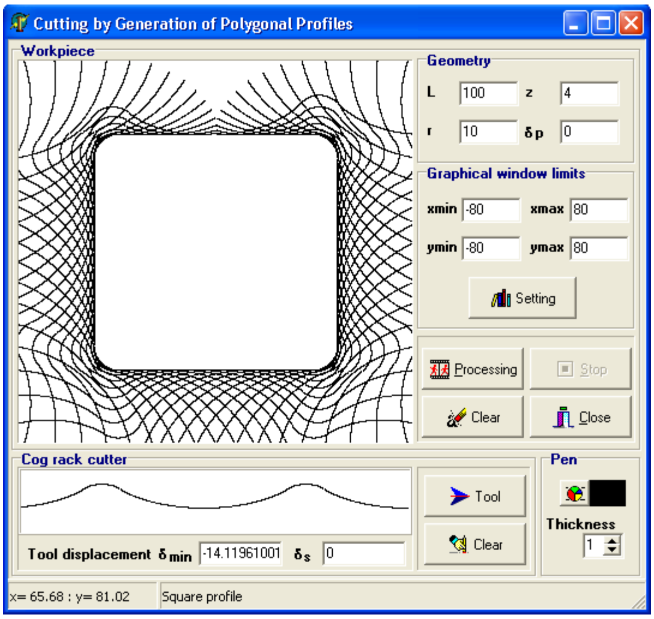


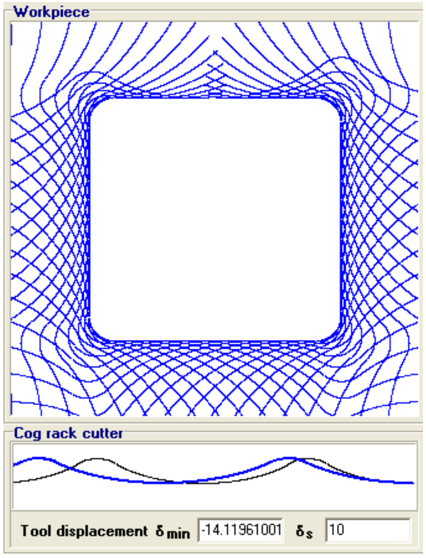

Figure 4. The interface of the simulation program for cutting by generation of the polygonal profiles

The main window is represented by the graphical window that allows the viewing of machining process.

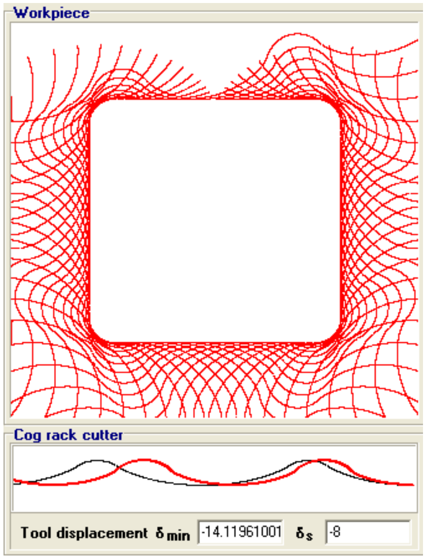
*Geometry* panel allows the setting of geometrical parameters like:

- L: the length of the side of polygonal profile;
- r: the rounding radius of the polygonal profile;
- z: number of sides of the polygonal profile;
- $\delta_p$: value of the displacement of tool towards the workpiece.

*Graphical window limits* panel allows setting of the minimum and maximum desired values corresponding to abscissa and ordinate (the zone that presents interest can be brought into the graphical window).

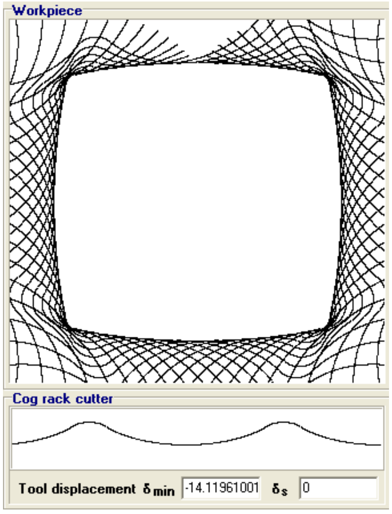



a. positive profile displacement of the tool $\delta_s = 10$

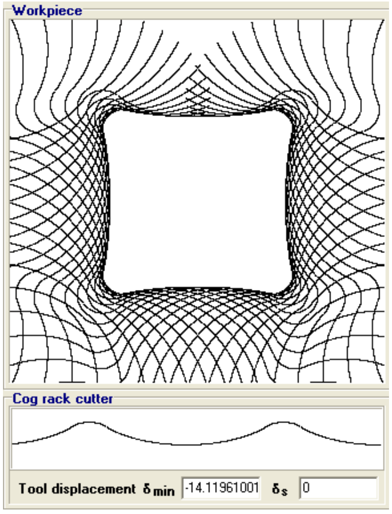



b. negative profile displacement of the tool $\delta_s = -8$

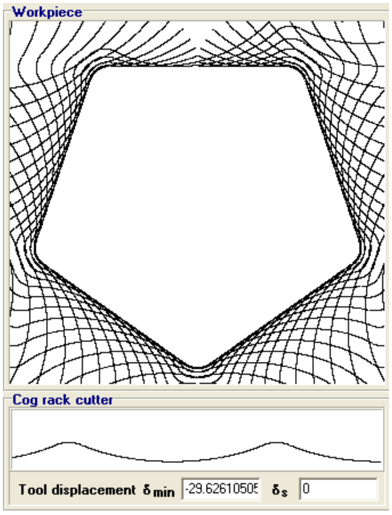
Figure 5. Simulation of processing of the same square profile with tools with different geometry



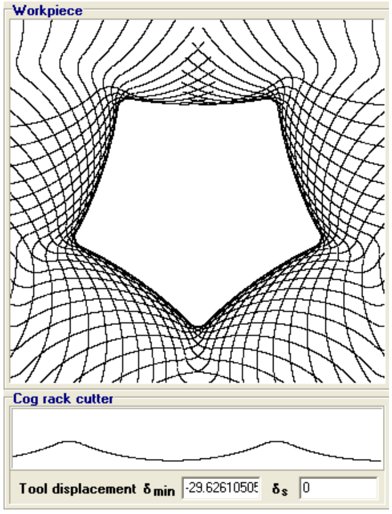

a. square profile, positive displacement $\delta_p$=12



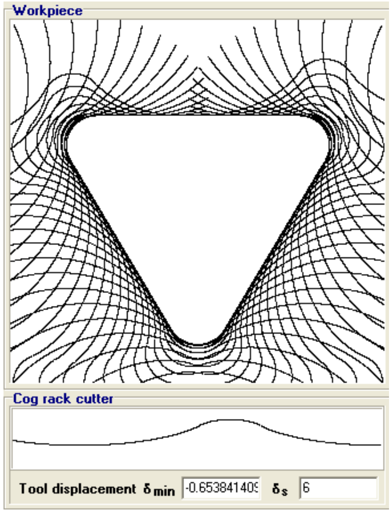

b. square profile, negative displacement $\delta_p$=-12



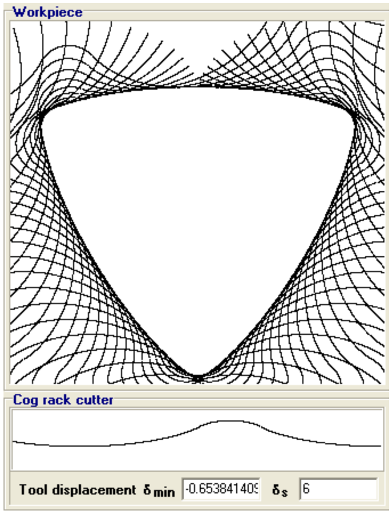

c. pentagonal profile, displacement $\delta_p$=0




d. pentagonal profile, negative displacement $\delta_p$=-20




e. triangular profile, displacement $\delta_p$=0




f. triangular profile, positive displacement $\delta_p$=-10

Figure 6. Simulation of the processing with same tool of the workpieces with different profiles

Therefore to provide a greater flexibility for the designed program, this allows by the activation of *Setting* button, from right side of the display, the setting of the display manner of graphic screen for workpiece obtained after simulation, depending on the user's desire.

The buttons *Processing* and *Stop* allow the starting, respectively the stopping of simulation process. The *Clear* button allows the erasing of graphic window of workpiece, after the change of its limits, or for re-drawing of other simulation situations, after the change of desired geometric parameters.

The *Cog rack cutter* panel contains a graphical window for visualization of the tool geometry, the *Tool* button that allows the plotting of tool profile and the *Clear* button for the erasing of graphic window of tool. The two text areas are for displaying the minimum value of tool displacement and the input of the desired value for tool displacement.

The *Pen* panel enables both the setting of thickness and color for each simulation.

The program status bar allows reading of the coordinates of interest points on the graphic screen, by focusing them with the mouse.

It should be noted that the processing of a particular workpiece profile may be achieved using cog rack cutter with different profiles, whose equations depend on the profile displacement, $\delta_s$. So the displacement, $\delta_s$, represents an important factor of geometry optimization of tools' profile (Fig. 5).

At the same time using the same cog rack cutter, by its rolling on circles of different diameters (modifying the displacement value, $\delta_p$) profiles of different pieces can be obtained. In this regard, in Fig. 6, it can noticed the possibility of obtaining "convex" profiles (positive values of displacement $\delta_p$) or "concave" profiles (negative values of $\delta_p$) for the desired workpieces, simply by changing the radius of rolling circle for the tool during machining.

## IV. Conclusions

The paper proposed the modeling and the designing of a cog rack cutter for continuous processing of the polygonal profiles, the method also extending the processing possibilities to polygonal profiles with odd number of sides.

At the same time the cutting by generation method allows a superior operational efficiency regarding the processing accuracy as well as the dimensions range of the manufactured profiles.

Following the suggested simulations by this paper, the conclusions below can be worded.

- The proposed study emphasized that the displacement ($\delta_s$) is a very important parameter for the optimization of tool profile.
- Using the cutting by generation process may be processed the polygonal surfaces both with an even number of sides, and polygonal surfaces with odd number of sides.
- The proposed solution, for generation of polygonal profiles, has clear benefits under the conditions of series production, where the tools' realization is economically justified. Also, the cutting by generation method is more accurate.

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