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Implementation of Mathematical Model of Window Function for Designing a Symmetrical Low Pass FIR Filter

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Abstract- Low pass filter is very important part of Digital Signal Processing. Taking Fourier Transform of ideal filter kernel (impulse response) gives ideal low pass filter. But when taking Fourier Transform of truncated *sinc* filter kernel then it gives frequency response near to ideal low pass filter with ripple in passband and stopband at frequency domain. Transition band between passband and stopband also wider than ideal low pass filter. To remove this ripple and to get a better transition band at frequency domain, here we design and implement a symmetrical low pass FIR filter at frequency domain by window technique and compare the performance of our proposed window function with the existing window function.

Keywords- *Low Pass Filter, Frequency impulse response filter, Sinc function, Window Function*

I. INTRODUCTION

The fastest field of development of modern electronic engineering is the field of digital signal processing. Enormous demand for telecommunications equipment, real-time data acquisition and processing and the quest for ever improving voice and video compression logarithms has fuelled research and development in the field of dedicated Digital Signal Processing (DSP) hardware and software [2]. In the last decade signal filtering and processing debuted into domains of business and economics, analyzing complex patterns of global financial movements and trying to predict future developments. This new generation of filters relies on advance mathematical models to quickly and precisely filter the signal and extrapolate any required predictions of future data [5]. A fundamental aspect of signal processing is filtering. Filtering involves the manipulation of the spectrum of a signal by passing or blocking certain portions of the spectrum, depending on the frequency of those portions. Filters are designed according to what kind of manipulation of the signal is required for a particular application.

Taking the Inverse Fourier Transform of ideal frequency response of low pass filter produces the ideal filter kernel (impulse response). This means that when taking Fourier Transform infinite length of ideal filter kernel or sinc function (impulse response) gives ideal low pass filter. But when taking Fourier Transform of truncated sinc filter kernel then it gives frequency response near to ideal low pass filter, because there are ripple in passband and stopband. Transition band between passband and stopband also wider than ideal low pass filter. To remove this ripple and to get a better transition band some Window technique is adopted.

The remainder of this paper is organized as follows. Section II describes the literature review and methodology. Section III describes performance evaluation of the proposed window. Finally, discussion and conclusion is presented.

II. LITERATURE REVIEW AND METHODOLOGY

In this paper we represent the design and implementation of a symmetrical low pass FIR filter at frequency domain by window technique, simulation result of the proposed mathematical model and compare the performance of our proposed window function with the existing window function.

At first we work on sinc function in MATLAB. We first draw truncated sinc function by using MATLAB program. Then shift truncated sinc function in time domain in left side by 5 scale and then shift in right side in 5 scale. Finally, we add the truncated sinc function and the two shifted sinc function. Then we implement a mathematical model corresponding to this resultant sinc function. After that we multiply the frequency response of the truncated sinc function with resultant sinc function after adding. Then we compare our proposed function with the existing Window function.

A. Finite Impulse Response Filter

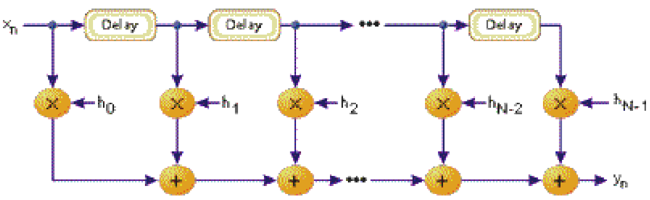

Figure 1. Finite Impulse Response (FIR) Filter of length N.

A finite impulse response (FIR) filter is a filter structure that can be used to implement almost any sort of frequency response digitally. An FIR filter is usually implemented by using a series of delays, multipliers, and adders to create the filter's output. Figure 1 shows the basic block diagram for a finite impulse response (FIR) filter of length N. The general form of Linear Time Invariant FIR system's output y[k] at time k is given by

$$y[k] = \sum_{n=0}^{N-1} h(n)x(k-n)$$

Where h(n) is the system's impulse response. The above equation indicates the output is a linear combination of present input and the N previous inputs.

B. FIR Coefficient Calculation Methods:

The important part of the design process is determining the filter coefficients. The most popular methods for FIR filter design are:

a) Window Method, b) Frequency Sampling Method, c) Optimal Filter Design.

C. SINC Function

The impulse responses (ideal filter kernel) of linear phase FIR filters are composed of sinusoids that exponentially decay in amplitude [3]. In principle, this makes their impulse responses infinitely long which ideal low-pass filter in frequency domain shown in Figure 2.

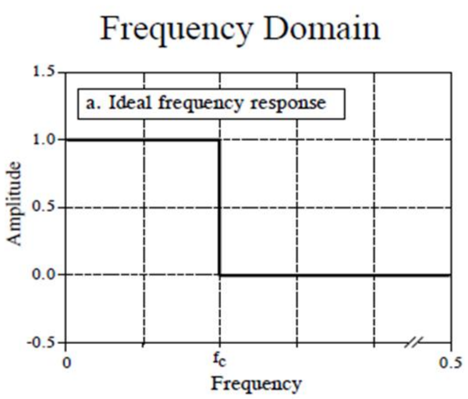

Figure 2. Frequency response of ideal filter kernel

By taking the Inverse Fourier Transform of ideal frequency response of low pass filter, produces the ideal filter kernel (impulse response) shown in Figure 3

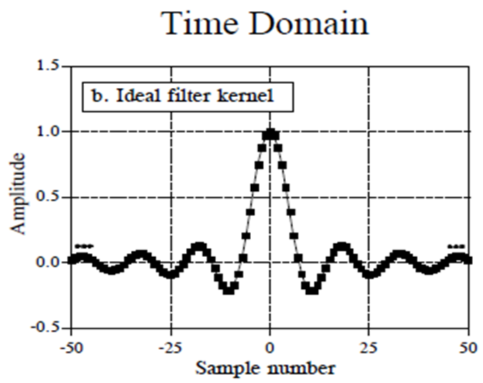

Figure 3. Impulse response of ideal filter kernel

Figure 3 is a Sinc Function and is given by:

$$h[i] = \frac{sin(2\pi fci)}{i\pi}$$

D. Window Method

The basic idea behind the window method is to choose a proper ideal frequency-selective filter (which always has a non-causal, infinite-duration impulse response) and then truncate (or window) its impulse response to obtain a linear-phase and causal FIR filter. Therefore, the emphasis in this method is on selecting an appropriate windowing function and an appropriate ideal filter.

E. Several Window Function

Several Window function are given below-

a) Blackman Window:

$$w(n) = 0.42 - 0.5\cos\left(\frac{2\pi n}{M-1}\right) + 0.08\cos(\frac{4\pi n}{M-1})$$

b) Hamming Window:

$$w(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{M-1}\right) \qquad [1]$$

c) Hanning Window:

$$w(n) = 0.5 - 0.5\cos(\frac{2\pi n}{M-1})$$

III. PERFORMANCE EVALUATION OF THE PROPOSED WINDOW

A) MATLAB Simulation of Sinc Function

We first draw sinc function by using MATLAB program shown in Figure 4. Then shift sinc function in time domain in right side by 5 scales shown in Figure 5 and then shift in left side in 5 scales shown in Figure 6. Finally we add the sinc function and the two shifted sinc function shown in Figure 7, which is our proposed window.

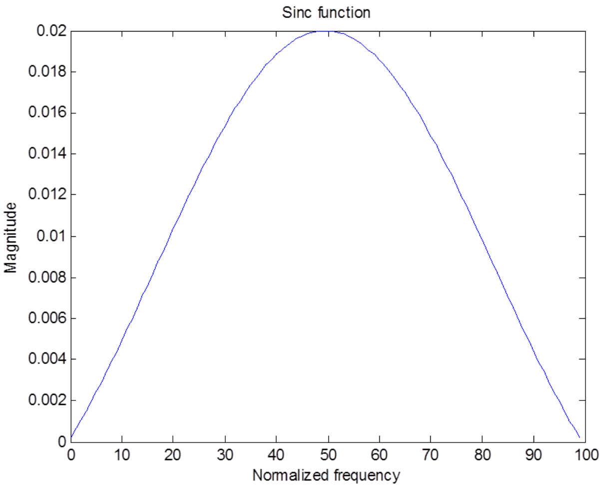

Figure 4. SINC Function

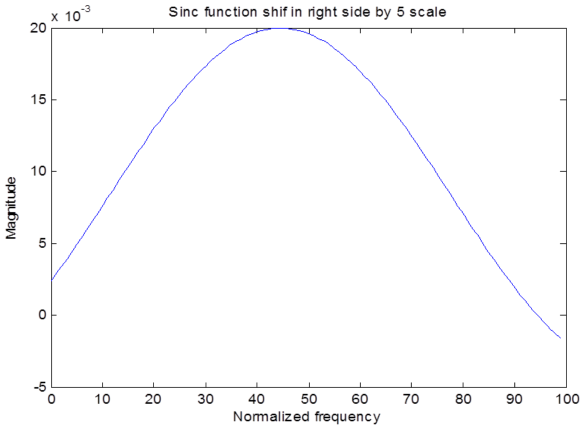

Figure 5. Sinc function shifted right side by 5 scale

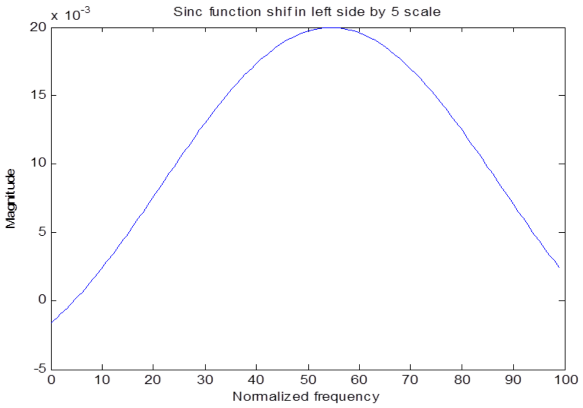

Figure 6. Sinc function shifted leftt side by 5 scale

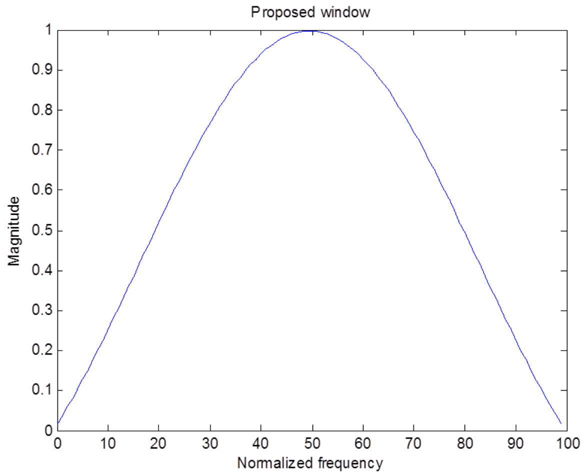

Figure 7. Sum of three sinc function or proposed window

B) Mathematical Expression for Proposed Window

Mathematical expression of proposed window is the sum of three sinc function that is, one is the original sinc function, two are the shifted sin function- one is shifted in right side by 5 scale and the other is shifted in left side by 5 scale. Derivation of mathematical expression of proposed window is given below:

Let, us assume f_c is the cutoff frequency, i is the sample number then $\omega_c = 2\pi f_c$

Mathematical expression of sinc function is,

$$sinc1 = \frac{sin\omega_c\, n}{\pi n}$$

Now, shifted sinc function in right side by 5 scale is,

$$sinc2 = \frac{sin\omega_c\ (n+5)}{\pi(n+5)}$$

and shifted sinc function in left side by 5 scale is,

$$\mathrm{sinc3} = \frac{sin\omega_c\ (n-5)}{\pi(n-5)}$$

Now, sum of the three sinc function is,

w(n)= sinc1 + sinc2 + sinc3

$$= \frac{sin\omega_c\, n}{\pi n} + \frac{sin\omega_c\ (n+5)}{\pi(n+5)} + \frac{sin\omega_c\ (n-5)}{\pi(n-5)}$$

Which, results in-

$$w(n) = \frac{sin\omega_c\, n}{\pi n} + \left\{\frac{1.902113n}{\pi(n^2-25)}\right\} sin\omega_c n - \left\{\frac{3.090169}{\pi(n^2-25)}\right\} cos\omega_c n$$

C) Multiplication with Impulse Response of Sinc Function

After designing and implementing of proposed window function, we multiply it with frequency response of truncated sinc function and the resultant frequency response is shown in Figure 8. Truncated sinc function and corresponding frequency response is shown in Figure 9, Figure 10 and Figure 11 respectively.

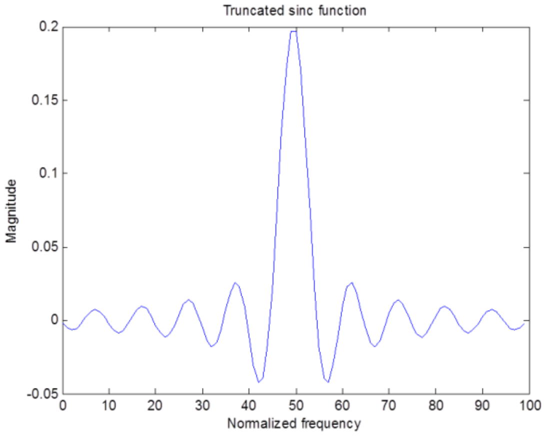

Figure 8. Truncated sinc Function

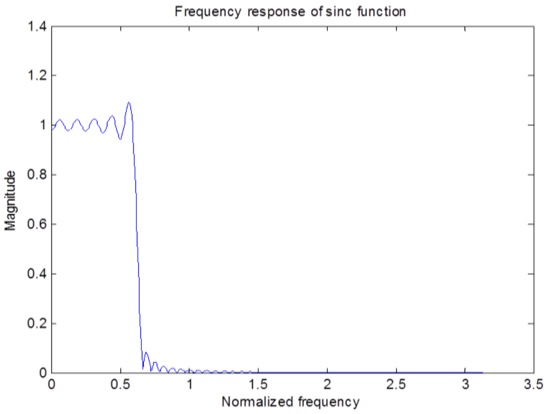

Figure 9. Frequency response of truncated sinc function

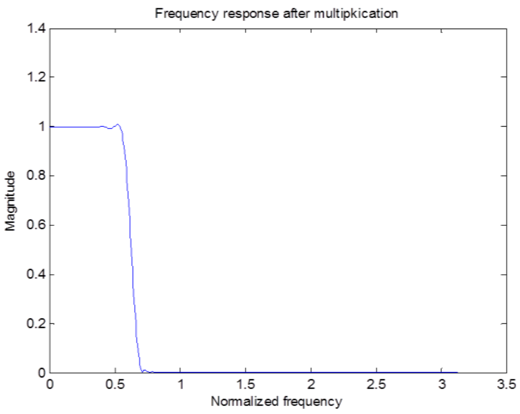

Figure 10. Frequency response after multiplication of proposed window

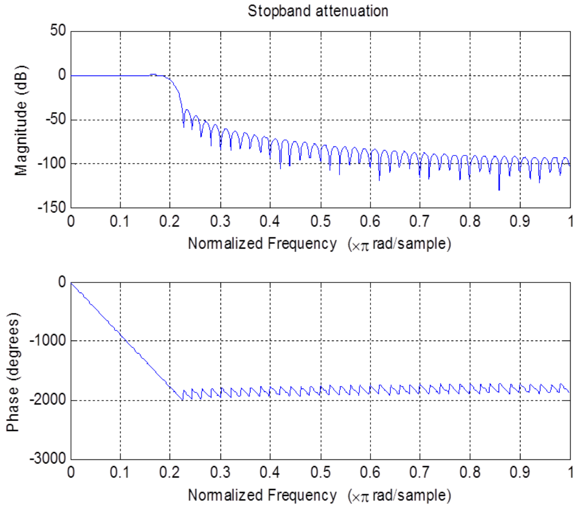

Figure 11. Stopband attenuation after multiplication with proposed window

IV. RESULT

Performance evaluation of our proposed window function and other several window functions at different values of filter length kernel M (=50, 76, and 100) is shown in tabular form below:

TABLE I. COMPARISON AMONG PROPOSED WINDOW AND OTHER SEVERAL WINDOWS

Name of the Window	Filter length Kernel M	Passband Ripple (f)	Transition Band (f)	Stopband Attenuation (dB)
Proposed window	50	0.8215	0.0606	-26.48
	76	0.2752	0.0488	-32.59
	100	0.143	0.0528	-38.67
Blackman Window	50	0.0086	0.1092	-75.54
	76	0.0086	0.1172	-75.16
	100	0.0086	0.0899	-75.36
Hamming Window	50	0.0476	0.1348	-51.33
	76	0.0338	0.0898	-55.57
	100	0.1634	0.0684	-55.58
Hanning Window	50	0.0692	0.1328	-43.98
	76	0.0674	0.0839	-43.95
	100	0.0683	0.0644	-43.95

From our study, we were able to achieve the following points:

- We can reduce passband ripple and stopband attenuation in frequency domain.
- By comparing other existing window function we see that our proposed window function have better transition bandwidth in frequency domain than others. Which ensure a fast roll-off.

The application of this study may be implemented in the following systems- Communications systems, Audio systems such as CD players, Instrumentation, Image processing and enhancement, Processing of seismic and other geophysical signals, Processing of biological signals, Artificial cochlea, Speech synthesis etc..

V. CONCLUSION

A symmetrical low pass linear-phase finite impulse response filter can play a vital role in the field of digital signal processing. The reason behind this is that every filter (including high pass, band pass) can be implemented with the help of low pass filter. The window technique is best described in terms of designing a symmetrical low pass linear-phase FIR filter. If our proposed window have better performance in passband and stopband by farther research in frequency domain then it would be one of the most straightforward and popular method for designing low pass filter.

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