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Length Effect on the Damping of Unidirectional Beams

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Abstract- The paper presents an evaluation of the damping prediction of unidirectional composite using the finite element method, which takes account the effect of the beam length. This study follows the evolution of the damping when the length increases by using modal analysis with different application of load rate at the structure. An analytical method is used to solve the equation of free vibrations. The study shows the decrease in frequency for different rates of loading, hence the loss of stiffness for all beams studied. The calculation of loss factors of modal energies for the first three bending modes of the structure is done by evaluating the ratio of the strain energies of beam for damaged and undamaged cases. The structural damping of the different beams is evaluated from these energies.

Keywords- *Damping; finite element method; frequency; unidirectional composite; length.*

I. Introduction

Damping is a measure of the energy dissipation in any vibrating structure. The progress has been achieved in the analysis and measurement of dynamic properties of composite materials. The initial works on the damping analysis of fibre composite materials were reviewed extensively in review paper by Berthelot, Gibson and Plunkett [1-2] and Gibson and Wilson [3]. A damping process has been developed initially by Adams and Bacon [4] who sees that the energy dissipation can be described as separable energy dissipations associated to the individual stress components. This analysis was refined in later paper of Ni and Adams [5]. The damping of orthotropic beams is considered as function of material orientation and the papers also consider cross-ply laminates and angle-ply laminates, as well as more general types of symmetric laminates.

The damping concept of Adams and Bacon was also applied by Adams and Maheri [6] to the investigation of angle-ply laminates made of unidirectional glass fibre or carbon layers. The finite element analysis has been used by Lin et al. [7] and Maheri and Adams [8] to evaluate the damping properties of free-free fibre reinforced plates. These analyses were extended to a total of five damping parameters, including the two transverse shear damping parameters. More recently the analysis of Adams and Bacon was applied by Yim [9] and Yim and Jang [10] to different types of laminates, then extended by Yim and Gillespie [11] including the transverse shear effect in the case of 0° and 90° unidirectional laminates. For thin laminate structures the transverse shear effects can be neglected and the structure behavior can be analyzed using the classical laminate theory.

The natural frequencies and mode shapes of rectangular plates are well described using the Ritz method introduced by Young [12] in the case of homogeneous plates. The Ritz method was applied by Berthelot and Safrani [13] to describe the damping properties of unidirectional plates. The analysis was extended to the damping analysis of laminates [14]. This paper presents an evaluation of the damping as function of the length using finite element method for a material with stacking sequence U.

II. Composite Material

The laminates were prepared by hand lay-up process from SR1500 epoxy resin with SD2505 hardener and unidirectional E-glass fibre fabrics of weight 300g/m2 . The evaluation of damping was performed on beams of different lengths: 140,160, 180, 200, 210, 220, 230 and 240 mm. Beams had a nominal width of 20 mm, were cured at room temperature with a pressure of 30 kPa using vacuum moulding process, and then post-cured for 8h at 80°C in an oven. Beams had a nominal thickness of 2 mm with a volume fraction of fibres equal to 0.40. The mechanical modulus of elasticity of the materials was measured in static tensile. The results are reported in table 1:

TABLE I. Mechanical Properties

Material	Stacking sequences	Young's modulus	Max load at fracture (KN)
U	$[(O)]_8$	21.08	35.165

The experimental investigation was conducted using tensile cyclic tests for different laminates studied. The applied load ratio is 10 % of maximum load failure. Fig.1 shows the results obtained for the Young's modulus reduction as a function of cycle number.

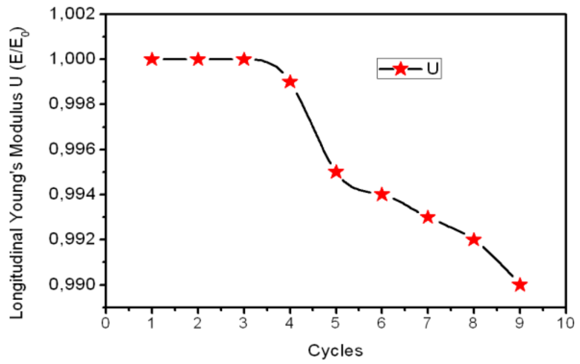

Figure 1. Stiffness reduction of unidirectional composite as a function of cycle number

III. Values Of Natural Frequencies

The differential equation of free motion for an undamped beam may be written [15] as:

$$\rho_s \frac{\partial^2 w_0}{\partial t^2} + k_s \frac{\partial^4 w_0}{\partial x^4} = 0 \qquad (1)$$

Where ρs is the mass per unit area and ks is the stiffness per unit area given by:

$$k_s = \frac{1}{D_{11}^{-1}} \qquad (2)$$

Eq. (1) of transverse vibrations may be rewritten in the form:

$$\frac{\partial^2 w_0}{\partial t^2} + \omega_0^2 L^4 \frac{\partial^4 w_0}{\partial x^4} = 0 \qquad (3)$$

Introducing the natural angular frequency of the undamped beam:

$$\omega_0 = \frac{1}{L^2}\frac{k_s}{\rho_s} = \frac{1}{L^2}\sqrt{\frac{1}{\rho_s D_{11}^{-1}}} = \frac{1}{L^2}\sqrt{\frac{E_x h^3}{12\rho_s}} \qquad (4)$$

The angular frequency of mode (i) is given by:

$$\omega_i = \lambda_i^2 \omega_0 \qquad (5)$$

The coefficient λi are reported in table II.

TABLE II. Values Of The Coefficients Of The Clamped-Free Beam Function

i	1	2	3	4	5	6	7
λi	1.8751	4.6941	7.8548	10.996	14.137	17.279	20.420

IV. Finite Element In The Dynamic Analysis

The flexural vibrations of beams are analyzed by the finite element method, using the stiffness matrix and mass matrix of beam element with two degrees of freedom per node (Fig.2):

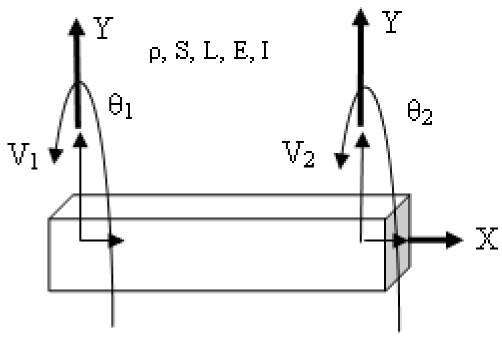

Figure 2. Element beam

Where:

E: the Young modulus.

I: the moment of inertia of the beam.

L: the length of the beam.

S: the section of the beam.

ρ: the density.

$$K_e = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \qquad (6)$$

$$M_e = \frac{\rho S L}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \qquad (7)$$

The global matrix of mass and stiffness are obtained by using assembly method:

$$K_G = B^T K_{des} B$$
$$M_G = B^T M_{des} B \qquad (8)$$

Where:

- B is the Boolean matrix.
- K_{des} and M_{des} are unassembled matrix, they contain only elementary matrix of mass and stiffness.

$$K_{des} = \begin{bmatrix} [K_e^{\,1}] & 0 \\ 0 & \ddots [K_e^{\,N}] \end{bmatrix} \qquad (9)$$

$$M_{des} = \begin{bmatrix} [M_e^{1}] & 0 \\ 0 & \ddots [M_e^{N}] \end{bmatrix} \quad (10)$$

The equation of motion [20]:

$$m\ddot{q}(t) + kq(t) = 0 \quad (11)$$

The equation (11) can be written in matrix form:

$$[M]\{\ddot{q}\} + [K]\{q\} = 0 \quad (12)$$

With q is the vector of degrees of freedom. We have two cases where the structure is undamaged [K] = [K_G] and damaged [K] = [K_G^D] which takes into account the decrease in the rigidity of the structure when the loading rates change [17].

The general solution of equation (12) is:

$$\{q\} = \{q_0\}\, e^{i\omega t} \quad (13)$$

By substituting the equation (13) in equation (12), we have:

$$[K]\{q_0\} = \omega^2\, [M]\{q_0\} \quad (14)$$

Then, the determinant must be zero:

$$det([K] - \omega^2 [M]) = 0 \quad (15)$$

There are many methods to calculate the eigenvalues; the most of these methods are to write the equation (11) as follows:

$$[H]\{X\} = \lambda\, \{X\} \quad (16)$$

Where [H] is a positive and symmetric matrix, it is clear that if we write directly the equation (14) as:

$$[K]^{-1}\, [M]\{q_0\} = \frac{1}{\omega^2}\, \{q_0\} \quad (17)$$

Where $[K]^{-1}$ is the inverse of the matrix [K], the symmetry property is not always preserved. Therefore, it is necessary to write the matrix [K] using the Cholesky decomposition:

$$[K] = [L]\,[L]^T \quad (18)$$

$[L]^T$ is the transpose of the matrix [L] and [L] is a lower triangular matrix.

The equation (14) is written:

$$[L]^{-1}\, [M]\,[L]^{-T}\, [L]\{q_0\} = \frac{1}{\omega^2}\, [L]\{q_0\} \quad (19)$$

By writing equation (19) as similar form as equation (14):

$$[H] = [L]^{-1}\, [M]\,[L]^{-T} \quad (20)$$

$$\{X\} = [L]\{q_0\} \quad (21)$$

$$\lambda = \frac{1}{\omega^2} \quad (22)$$

V. Evaluation Of The Beam Damping As Function Of Lengths

The modal strain energy of the beam for the undamaged case [17-18] is given by:

$$U_n = \frac{1}{2}[\phi_n]^T\, [K_G]\, [\phi_n] \quad (23)$$

The modal strain energy for damaged case is given by:

$$U_{nD} = \frac{1}{2}[\phi_{nD}]^T\, [K_G^{\,D}]\, [\phi_{nD}] \quad (24)$$

With [$Ø_n$], [$Ø_{nD}$] are the eigenvectors of displacement for undamaged and damage case.

The loss factor coefficient [18] for different stages of damage (different loading rates) is given by:

$$\eta_n = \frac{\Delta U_n}{U_n} = \frac{U_n - U_{nD}}{U_n} \quad (25)$$

With U_n, U_{nD} are the modal strain energies for undamaged and damage case.

VI. Results And Discussions

The decrease in frequency of different loading rates shows the loss of stiffness for the height lengths of beams studied which constitute the method to follow the progression of fatigue damage of the composites [16, 19].

TABLE III. Frequencies Obtained By The Model And Analytical Method

Length of beam (m)	Analytical Response (Hz)	Modeling frequencies with Different loading rates (Hz)		
		0 %	*50 %*	*90 %*
L1=0.14	38.453	36.684	36.592	36.5
	346.08	352.12	351.24	350.36
	961.33	933.16	930.83	928.49
L2=0.16	29.441	28.086	28.016	27.945
	264.97	269.59	268.92	268.24
	736.02	714.45	712.67	710.87
L3=0.18	23.262	22.192	22.136	22.08
	209.36	213.01	212.48	211.95
	581.55	564.51	563.09	561.68
L4=0.20	18.842	17.975	17.93	17.885
	169.58	172.54	172.11	171.68
	471.05	457.25	456.11	454.96

The programming of this resolution method was performed under the Matlab software. The frequencies obtained by the model and the analytical response for different lengths studied are reported in tables III and IV.

TABLE IV. FREQUENCIES OBTAINED BY THE MODEL AND ANALYTICAL METHOD

Length of beam (m)	Analytical Response (Hz)	Modeling frequencies with Different loading rates (Hz)		
		0 %	*50 %*	*90 %*
L5=0.21	17.09	16.304	16.263	16.222
	153.81	156.5	156.11	155.72
	427.26	414.74	413.7	412.66
L6=0.22	15.572	14.855	14.818	14.781
	140.15	142.6	142.24	141.88
	384.3	377.89	376.95	303.47
L7=0.23	14.247	13.592	13.558	13.524
	128.23	130.47	130.14	129.81
	356.18	345.75	344.88	344.01
L8=0.24	13.085	12.483	12.451	12.42
	117.76	119.82	119.52	119.22
	327.70	317.53	316.74	315.94

Figs.3-4 report the results deduced for the damping by finite element analysis for the first three modes. The evaluation of laminate damping by modeling takes account of the variation of the loss factor η with lengths. We have two cases:

- When the loading rate is 50 %: The Figure 3 shows an increase in damping (0.5025 %) when the length increases except for L4; we observe a slight decrease in damping (0.497 %).
- When the loading rate is 90 %: The Figure 4 shows an increase in damping (1.003 %) when the length increases except for L7; we observe a slight decrease in damping (0.997 %).

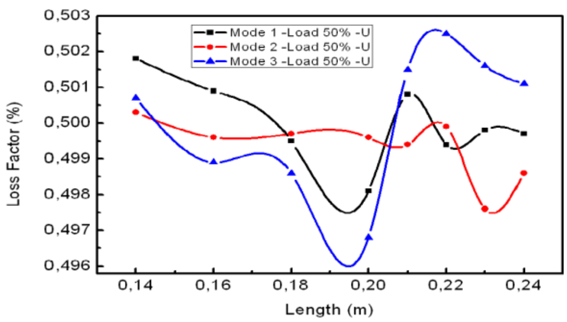

Figure 3. Modelling results obtained for the damping as function of the frequency for U material in the case: load 50 %.

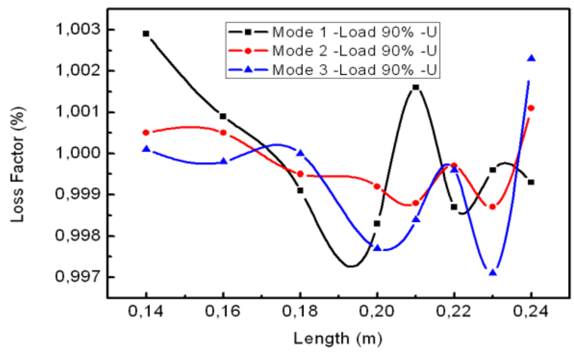

Figure 4. Modelling results obtained for the damping as function of the frequency for U material in the case: load 90 %.

VII. CONCLUSIONS

This study presents an evaluation of the damping of different lengths of composite material was presented based on a finite element analysis of the vibrations of a composite structure.

The decrease in frequency of different loading rates shows the loss of stiffness for the height lengths of beams studied which constitute the method to follow the progression of fatigue damage of the composites. The results deduced from the damping by finite element analysis for the first three modes that the evaluation of laminate damping takes account the variation of the loss factor η with lengths. The loss factors of the composite materials can be deduced by applying modeling to the flexural vibrations of free-clamped beams.

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