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Reconstruction of Variational Iterative Method for Solving Fifth Order Caudrey-Dodd-Gibbon (CDG) Equation

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Abstract-Engineering problems and the solutions of many important physical problems are centered on finding accurate solutions to Nonlinear functions. Various approximation methods have been used for complex equations. One of the newest approximation methods is Reconstruction of variational iteration method (RVIM). In this Paper we use RVIM to solution N-soliton solutions for the fifth order Caudrey-Dodd-Gibbon (CDG) Equation. Results compared with those of Adomian's decomposition method (ADM).The comparisons show that the Reconstruction of RVIM is very effective and overcome the tedious work that traditional methods require and quite accurate to systems of non-linear partial differential equations.

Keywords-*Reconstruction of Variational Iteration Method (RVIM), Caudrey-Dodd-Gibbon (CDG) Equation, Adomian decomposition method (ADM).*

I. INTRODUCTION

In recent years the theory of solitary waves has attracted much concern for treatment of PDEs describing nonlinear and development concepts. Partial differential equations which emanate in real-world physical problems are often too complicated to be solved exactly, and even if an exact solution is obtainable, such as inverse scattering method, Ba¨cklund transformation method, Hirota's bilinear scheme [1-2], Hereman's method [3], pseudo spectral method, Jacobi elliptic method, Painleve´ analysis [4], and other methods, the required calculations may be practically too complicated, or it might be difficult to expound the outcome. With the rapid promotion of linear and nonlinear science, in the past several decades, various methods for obtaining solutions of DEs have been presented, such as Homotopy perturbation method, variational iteration method, exp-function method and RVIM and so on.

RVIM has been shown to solve a large class of nonlinear problems with approximations converging to solutions rapidly, effectively, easily, and accurately. besides the aim of this paper is to show that RVIM is strongly and simply capable of solving a large class of linear or nonlinear differential equations without the tangible restriction of sensitivity to the degree of the nonlinear term. The most sensible advantages of RVIM are using Laplace Transform and choosing initial conditions simply and easily in solving linear and nonlinear equations.
In this article we use RVIM to solve the N-soliton solutions for the fifth order Caudrey-Dodd-Gibbon (CDG) Equation.

$$u_t + u_{xxxxx} + 30uu_{xxx} + 30u_x u_{xx} + 180u^2 u_x = 0 \quad (1)$$

with $u(x,t)$ is a adequately often differentiable function. The CDG equation dominate the Painleve´ property as demonstrated by Weiss in [4]. A useful study is introduced in [4] using the Painleve´ property and the Ba¨cklund transformation in handling the CDG equation and other equations as well. It was found in [4] that the CDG Equation (1) has the Backlund transformation

$$u = \frac{\partial^2}{\partial x^2} \ln \Phi + u_2 \quad (2)$$

where u_2 satisfies the CDG equation, and

$$u_2 = -\frac{1}{6}\frac{\Phi_{xxx}}{\Phi_x}$$

and

$$\frac{\Phi_t}{\Phi_x} + \frac{\partial^2}{\partial x^2}\{\Phi; x\} + 4\{\Phi; x\}^2 = 0$$

The last two equations can be expressed as the Lax pair

$$\Phi_{xxx} + 6u_2\Phi_{xxx} = 0$$

$$\Phi_t + 18u_{2x}\Phi_{xx} + 6(6u_2^2 - u_{2xx})\Phi_x = 0$$

The objective of this work is to promote other studies related to the CDG equation. The tanh method [5, 6], and the tanh-coth method [7] will be used to emphasize its power in the determination of single-soliton solution and other travelling wave solutions. We plan to use RVIM to solve this equation.

II. Basic Idea of RVIM

To clarify the basic ideas of our proposed method in [8], we consider the following differential equation same as VIM based on Lagrange multiplier [9]:

$$Lu(x_1,\cdots,x_k)+Nu(x_1,\cdots,x_k)=f(x_1,\cdots,x_k) \tag{3}$$

By suppose that

$$Lu(x_1,\cdots,x_k)=\sum_{i=0}^{k}L_{xi}u(x_i) \tag{4}$$

where L is a linear operator, N a nonlinear operator and $f(x_1,...,x_k)$ an inhomogeneous term.

we can rewrite equation (3) down a correction functional as follows:

$$L_{xj}u(x_j)=\underbrace{f(x_1,\cdots,x_k)-Nu(x_1,\cdots,x_k)-\sum_{\substack{i=0\\ i\neq j}}^{k}L_{xi}u(x_i)}_{h((x_1,\cdots,x_k),u(x_1,\cdots,x_k))} \tag{5}$$

therefore

$$L_{xj}u(x_j)=h((x_1,\cdots,x_k),u(x_1,\cdots,x_k)) \tag{6}$$

With artificial initial conditions being zero regarding the independent variable x_j.

By taking Laplace transform of both sides of the equation (6) in the usual way and using the artificial initial conditions, we obtain the result as follows

$$P(s).U(x_1,\cdots,x_{i-1},s,x_{i+1},x_k)=H((x_1,\cdots,x_{i-1},s,x_{i+1},x_k),u) \tag{7}$$

Where P(s) is a polynomial with the degree of the highest derivative in equation (7), (the same as the highest order of the linear operator L_{x_j}). The following relations are possible;

$$\ell[\mathrm{h}]=\mathrm{H} \tag{8-a}$$

$$\mathrm{B(s)}=\frac{1}{\mathrm{P(s)}} \tag{8-b}$$

$$\ell[\mathrm{b}(\mathrm{x_i})]=\mathrm{B(s)} \tag{8-c}$$

Which that in equation (8-a) the function $H((x_1,\cdots,x_{i-1},s,x_{i+1},x_k),u)$

and $h((x_1,\cdots,x_{i-1},x_i,x_{i+1},x_k),u)$

have been abbreviated as H, h respectively.

Hence, rewrite the equation (7) as;

$$U(x_1,\cdots,x_{i-1},s,x_{i+1},x_k)=H((x_1,\cdots,x_{i-1},s,x_{i+1},x_k),u).B(s) \tag{9}$$

Now, by applying the inverse Laplace Transform on both sides of equation (9) and by using the (8-a) - (8-c), we have;

$$\mathrm{u}(\mathrm{x}_1,\cdots,\mathrm{x}_{i-1},\mathrm{x}_i,\mathrm{x}_{i+1},\mathrm{x}_k)=\int_0^{\mathrm{x}_i}\mathrm{h}((x_1,\cdots,x_{i-1},\tau,x_{i+1},x_k),u).b(x_i-\tau)d\tau \tag{10}$$

Now, we must impose the actual initial conditions to obtain the solution of the equation (3). Thus, we have the following iteration formulation:

$$\mathrm{u}_{n+1}(\mathrm{x}_1,\cdots,\mathrm{x}_{i-1},\mathrm{x}_i,\mathrm{x}_{i+1},\mathrm{x}_k)=\mathrm{u}_0(\mathrm{x}_1,\cdots,\mathrm{x}_{i-1},\mathrm{x}_i,\mathrm{x}_{i+1},\mathrm{x}_k)+\int_0^{\mathrm{x}_i}\mathrm{h}((x_1,\cdots,x_{i-1},\tau,x_{i+1},x_k),u).b(x_i-\tau)d\tau \tag{11}$$

Where u_0 is initial solution with or without unknown parameters. Assuming u_0 is the solution of *Lu*, with initial/boundary conditions of the main problem, In case of no unknown parameters, u_0 should satisfy initial/ boundary conditions. When some unknown parameters are involved in u_0, the unknown parameters can be identified by initial/boundary conditions after few iterations, this technology is very effective in dealing with boundary problems. It is worth mentioning that, in fact, the Lagrange multiplier in the He's variational iteration method is $\lambda(\tau)=b(x_i-\tau)$ as shown in [10].

The initial values are usually used for selecting the zeroth approximation u_0. With u_0 determined, then several approximations u_n $n>0$, follow immediately. Consequently, the exact solution may be obtained by using

$$\mathrm{u}(\mathrm{x}_1,\cdots,\mathrm{x}_{i-1},\mathrm{x}_i,\mathrm{x}_{i+1},\mathrm{x}_k)=\lim_{n\to\infty}\mathrm{u}_n(\mathrm{x}_1,\cdots,\mathrm{x}_{i-1},\mathrm{x}_i,\mathrm{x}_{i+1},\mathrm{x}_k). \tag{12}$$

III. Applying RVIM

Before To demonstrate the effectiveness of the method we consider here Eq.(1) with given initial condition.

Consider the Caudrey-Dodd-Gibbon (CDG) Equation (1) with the following initial condition:

$$u_0(x,t)=\mu^2\operatorname{sec}h^2(\mu x) \tag{13}$$

At first rewrite eq. (1) based on selective linear operator as

$$\ell\{u(x)\}=u_t=\overbrace{-(u_{xxxxx}+30uu_{xxx}+30u_xu_{xx}+180u^2u_x)}^{h(x,t,u)} \tag{14}$$

Now Laplace transform is implemented with respect to independent variable x on both sides of eq. (14) and by using the new artificial initial condition (which all of them are zero) we have

$$s \cup (x,t) = \ell\{h(x,t,u)\} \tag{15}$$

$$\cup (x,t) = \frac{\ell\{h(x,t,u)\}}{s} \tag{16}$$

And whereas Laplace inverse transform of *1/s* is as follows

$$\ell^{-1}[\frac{1}{s}] = 1 \tag{17}$$

Therefore by using the Laplace inverse transform and convolution theorem it is concluded that

$$u(x,t) = \int_0^t h(x,\varepsilon,u)d\varepsilon \tag{18}$$

Hence, we arrive the following iterative formula for the approximate solution of subject to the initial condition (13).

So, in exchange with applying recursive algorithm, following relations are achieved

$$u_{n+1} = u_0 + \int_0^t -(u_{xxxxx} + 30uu_{xxx} + 30u_x u_{xx} + 180u^2 u_x)d\varepsilon \tag{19}$$

Now we start with an arbitrary initial approximation $u_0(x,t) = \mu^2 \sec h^2(\mu x)$ that satisfies the initial condition and by using the RVIM iteration formula (19), we have the following successive approximation

$$u_1(x,t) = \frac{\mu^2(\cosh(\mu x) + 32t\mu^5 \sinh(\mu x))}{\cosh^3(\mu x)}$$

$$u_2(x,t) = \frac{1}{\cosh(\mu x)^{10}}\ \mu^2(\cosh(\mu x)^8 +$$

$$32\mu^5 \sinh(\mu x)t\cosh(\mu x)^7 + 163840\mu^{15}\sinh(\mu x)t^3\cosh(\mu x)^5$$

$$-614400\mu^{15}\sinh(\mu x)t^3\cosh(\mu x)^3$$

$$-7372800\mu^{20}t^4\cosh(\mu x)^2 + 2949120\mu^{20}t^4\cosh(\mu x)^4$$

$$+4423680\mu^{20}t^4 + 491520\mu^{15}\sinh(\mu x)t^3\cosh(\mu x)$$

$$+512\mu^{10}t^2\cosh(\mu x)^8 - 768\mu^{10}t^2\cosh(\mu x)^6$$

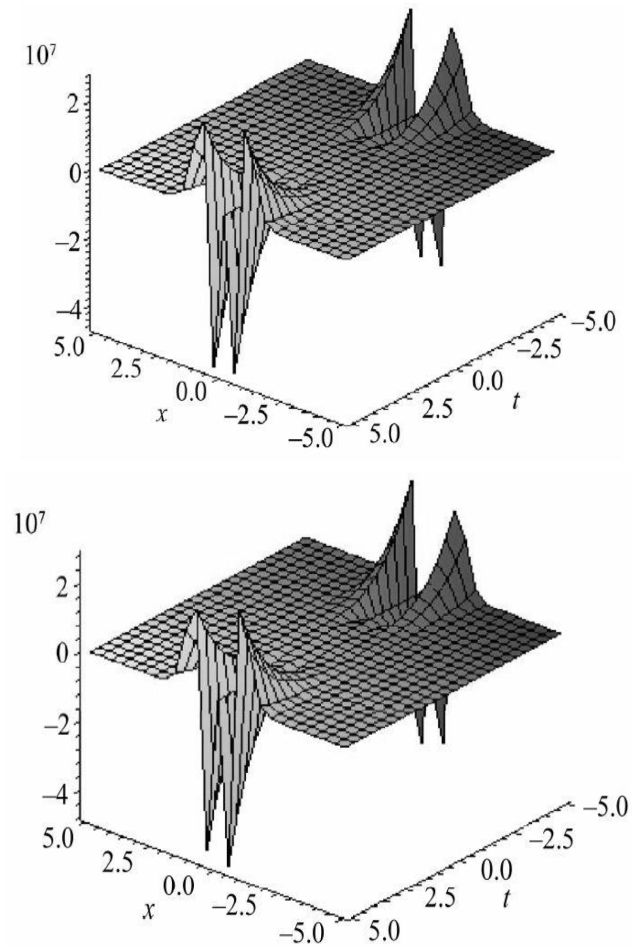

Figure.1. The surfaces on both columns respectively show the solutions, u(x,t), for RVIM on the top and ADM on the bottom when μ=1.

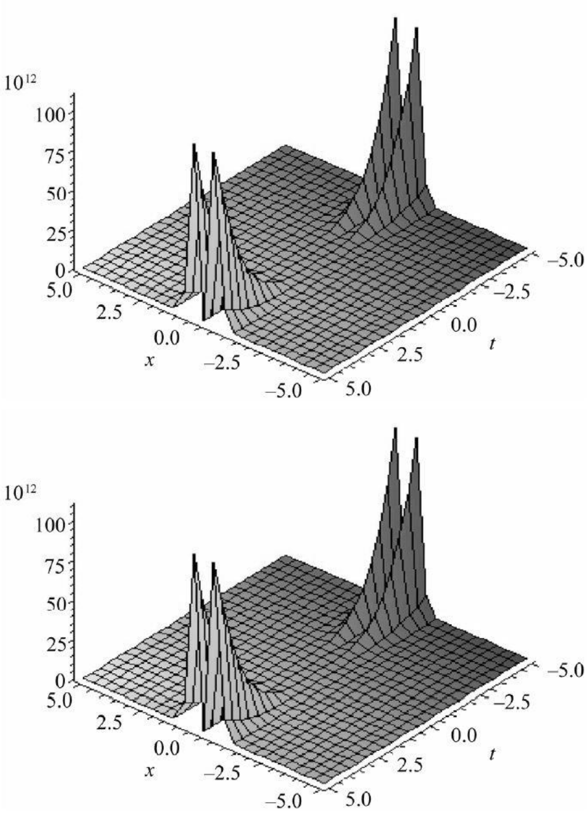

Figure.1. The surfaces on both columns respectively show the solutions, u(x,t), for RVIM on the top and ADM on the bottom when μ=2.

IV. CONCLUSION

In this paper, we successfully apply Reconstruction of Variational Iteration Method (RVIM) to approximate the solution of fifth order Caudrey-Dodd-Gibbon (CDG) Equation. Also, comparisons were made between He's variational iteration method and Adomian decomposition method (ADM) for Caudrey-Dodd-Gibbon (CDG) Equation. Moreover, the RVIM reduces the size of calculations by not requiring the tedious Adomian polynomials, and hence the iteration is direct and straightforward. The results reported here provide further evidence of the usefulness of RVIM for finding the analytic and numeric solutions for the linear and nonlinear diffusion equations and, it is also a promising method to solve different types of nonlinear equations in mathematical physics.

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