

# The Newton-Raphson Multivariate Analysis of a Water Distribution Network with Consideration of Minor Losses

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**Abstract**-This paper reexamines an earlier analysis of a water distribution network using a Newton-Raphson iterative method; this time considering the minor losses which were neglected in the earlier study. The network was intended to serve a two-wing student hostel with each wing having 5 building blocks each of which has 20 rooms. The sanitary appliances in each block consist of 6 showers, 8 lavatory sinks, 6 water closets with flush tanks and 2 hose bibs. The distribution network consists of 13 pipe sections. MatLab iterations were done using a Newton-Raphson multivariate method which resulted in an optimal solution set of pipe sizes, flow rates, and head losses relative to a reference starting node. It was observed that the minor losses in each pipe section increased the nodal head loss differential relative to the reference starting node. This work serves as a guide for the analyses of water networks of similar scope.

**Keywords**- Water Network Analysis, Newton-Raphson Method, Minor Losses, Regression Relations

## I. INTRODUCTION

Energy losses in water distribution pipes are accounted for as dissipation due to friction along the pipe lengths, and also due to pipe geometric variations and fittings mounted on the pipes. For long distance pipe networks, the frictional losses far outweigh the other forms of losses and as such are referred to major losses; while the others are categorized as minor. However, due to the multiplicity of fittings in many networks of practical relevance, the term ‘minor’ could be misleading.

Effects of frictional losses are expressible in the equations due to D’Arcy-Weisbach, Hazen-Williams, Hagen-Poiseuille and others, as well as the Moody diagram. On the other hand, minor losses are usually expressed as velocity heads which generate a formidable system of equations. In the present study, earlier results of regression models for determination of minor losses in water distribution systems are utilized, those models having been obtained from analyses using the aforementioned equations (Sodiki and Adigio, 2017a; Sodiki and Adigio, 2017b).

## II. SYSTEM DESCRIPTION

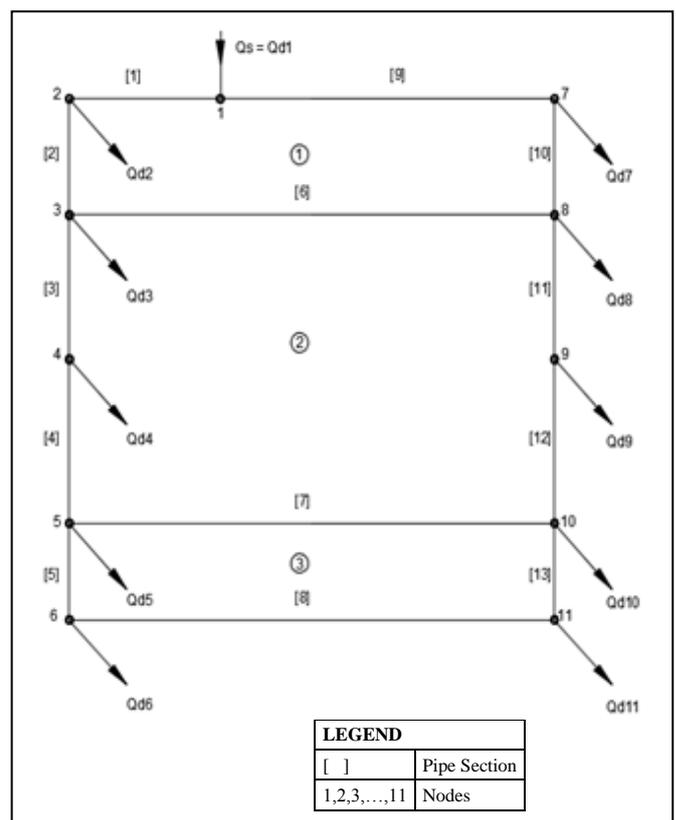


Figure 1. Schematic diagram of the proposed water distribution network

Field records of distances (hence pipe lengths) and sanitary appliance types and numbers were collated. The proposed distribution network which follows the architectural layout of the hostel is shown schematically in Fig. 1. In the figure, each discharge node supplies a block’s toilets and bathrooms. Thus, nodes 2 to 6 deliver to one wing while 7 to 11 deliver to the other wing. The delivery pipes are designated by the respective nodes at their ends.

The pipe lengths (i.e. the internodal distances) are presented in Table I. Supply into the distribution network from an elevated storage is done at Node 1 (i.e. the source node). A schedule – 40 polyvinyl chloride pipe material is utilized for the distribution.

TABLE I. PIPES AND THEIR LENGTHS

Pipes (designated number)	Pipes (node-node identification)	Lengths (mm)
1	1-2	37475
2	2-3	31918
3	3-4	31918
4	4-5	31918
5	5-6	31918
6	3-8	85950
7	5-10	85950
8	6-11	85950
9	1-9	48475
10	7-8	31918
11	8-9	31918
12	9-10	31918
13	10-11	31918

TABLE II. WATER SUPPLY FIXTURE UNITS (UNIFORM PLUMBING CODE, 2016)

Individual Fixtures	Minimum Fixture Branch Pipe Size		Water Supply Fixture Units (WSFU)	
	(inch)	(mm)	Private Installations	Public Installations
Bathtub	½	15	4	4
Bathtub with ¾" valve	¾	20	10	10
Bidet	½	15	1	
Dishwasher, domestic	½	20	1.5	1.5
Drinking fountain	½	15	0.5	0.5
Hose bib	½	15	0.5	2.5
Lavatory	½	15	1	1
Bar sink	½	15	1	2
Clinic faucet sink	½	15	3	
Kitchen sink, domestic	½	15	1.5	1.5
Laundry sink	½	15	1.5	1.5
Service or mop basin	½	15	1.5	3
Washup basin	½	15	2	
Shower head	½	15	2	2
Urinal with flush tank	½	15	2	2
Wash fountain	¾	20	4	
Water closet with gravity tank	½	15	2.5	2.5
Water closet with flushometer tank	½	15	2.5	2.5
Water cooler	½	15	0.5	0.5

In the computations, fixture units which account for the non-simultaneous use of all the installed sanitary appliances are assigned to each appliance (Table II). The units are 2 for a shower, 1 for a lavatory sink, 2.5 for a water closet and 2.5 for a hose bib. In the earlier study (Ifemi et al, 2020) the total discharge supplied from the tank to the network (through supply Node 1) which was calculated using these fixture units, and data for conversion to flow rates (Uniform Plumbing Code, 2016), was 0.01655m<sup>3</sup>/s. This value is also utilized in the present study.

### III. COMPUTATION PROCEDURES

The analysis of the network using fluid dynamics principles which had been elaborated in the earlier study (Ifemi et al, 2020) are outlined.

#### A. Limiting Velocity and Pressure Constraints

Rational lower and upper flow velocity limits had, respectively, been proposed by Kocyigit et al (2015) and Uniform Plumbing Code (2016) as 0.5m/s and 2.44m/s. These limits are adopted in the present study.

$$\text{Thus } 0.5\text{m/s} \leq V_j \leq 2.44\text{m/s}$$

where  $v_j$  = velocity of flow in pipe  $j$ ; for each  $j = 1, 2, \dots, NP$  and  $NP$  = number of pipes in the network.

Also, the residual pressure stipulated for the first index node in a water distribution system is 12.66mH<sub>2</sub>O (18psi) (Uniform Plumbing Code, 2016). This lower pressure limit is adopted in the present study.

#### B. Major Losses

The major losses are computed using D’Arcy-Weisbach equation expressed as (Ideriah, 2017)

$$H_{maj} = KQ^2 \tag{1}$$

$$\text{where } k = C_f \left(\frac{l}{D}\right) \left(\frac{32}{g\pi^2 D^4}\right)$$

$C_f$  is the friction factor,  $l$  is pipe length and  $D$  the pipe diameter.

$Q$  is the flow rate

To accommodate possible network solutions with some reversed (negative) flow directions, Eqn. 1 is modified as

$$H_{maj} = K |Q| \tag{2}$$

Thus, Eqn. 1 relates the major head loss, the pipe friction factor, pipe diameter and flow rate according to whether the flow is laminar, transitional or turbulent. The applicable equation for laminar flow is the Hagen – Poiseuille equation

$$C_f = \frac{16}{Re}; \text{ Re} < 2100 \tag{3}$$

where  $Re$  = flow Reynolds number

For the transitional and turbulent regimes the relevant equation is the Colebrook – White equation which combines the three parameters  $C_f$ ,  $Re$  and  $\bar{e}/D$  (which is the ratio of the average pipe roughness value  $\bar{e}$  to the pipe diameter  $D$ ) as (Douglas et al, 2011)

$$\frac{1}{\sqrt{C_f}} + 4 \log_{10} \left( \frac{\bar{e}/D}{3.71} + \frac{1.26}{Re \sqrt{C_f}} \right) = 0 \quad (4)$$

### C. Minor Losses

Generally, extensive runs of pipe result in increased frictional (major) loss, while a multiplicity of pipe fittings and changes in geometry of pipe cross-section is associated with increased minor loss. It is generally observed that for a given system configuration (for instance, for a water distribution system serving a group of buildings), the ratio between the total major loss and total separation loss for an index pipe run varies with varying length of run, and other system parameters (such as flow rate and number of sanitary appliances served). The dependence of the ratio on the length of run and other parameters is exemplified by the stipulation of SpiraxSarco Ltd (2017) of 10% of the major loss for most purposes but 30% for short pipes having a lot of fittings, to account for the minor loss in index runs. Also, in considering water distribution systems in buildings, Barry (1998) had considered it necessary to make an estimate of the likely length of pipe whose resistance to flow is equivalent to the resistance of all the pipe fittings (taken together) in the index run, as a percentage of the actual pipe length. In his opinion, this percentage might vary from 25 to over 100, which with experience would approach a fair degree of accuracy. Several others had suggested percentages to be added to the major loss in straight pipes to account for the minor loss due to all installed pipe fittings in index runs (Church, 1979; Fluid Handling Inc., 2008; Tiscala U. K. Ltd, 2013; Uponsor Plumbing Systems, 2017).

In all the above-mentioned instances of the percentages for approximating minor losses, no clear mathematical or statistical basis had been indicated. Furthermore, the variation of the major and minor loss components with varying system complexity (in terms of pipe length, flow rate and number of sanitary appliances, for instance) and, hence, varying percentages representing the minor loss had not been indicated. In order to address these shortcomings, studies had been done to develop regression model equations for approximating the minor loss as a fraction or percentage of the total head loss for varying system complexities (Sodiki and Adigio, 2017a; Sodiki and Adigio, 2017b). In those studies, data generated by the existing calculation methods such as the D'Arcy-Weisbach equation provide the basis for arriving at the useful approximations, through the regression analyses.

Now, in the present study, the total number of sanitary appliances in each building block is 44, the total number in each wing of 5 blocks is 220, and the total in the entire hostel of 2 wings is 440. The relevant regression equation relating number of appliances, denoted as  $x$ , with the fraction of the total loss which represents the minor loss, denoted as  $y$  is (Sodiki and Adigio, 2017b)

$$y = 0.157 + 0.0024x - 4 \times 10^{-6}x^2 \quad (5)$$

Substituting  $x = 440$ , gives  $y = 0.4386$ .

$$\text{Now, total head loss } H = H_{maj} + H_{min} \quad (6)$$

where  $H_{maj}$  is the major loss and  $H_{min}$  is the minor loss.

$$\therefore H_{min} = y H$$

then  $H = H_{maj} + y H$

$$H_{maj} = H - y H = H (1 - y)$$

and

$$H = \frac{H_{maj}}{1-y} \quad (7)$$

$$= 1.781 H_{maj}$$

### D. Pipe Size and Flow Determination

The procedure for determination of pipe sizes and flow rates for each pipe section of Fig. 1 using Newton – Raphson multivariable method had been elaborated in the earlier paper (Ifiemi et al, 2020). The flow chart of Fig. II illustrates the procedure.

## IV. RESULTS AND DISCUSSIONS

Using the available data, results were obtained from over 200 iterative network solutions using MatLab codes for each of the cases of negligible minor losses and of inclusion of minor losses, as elaborated in the earlier paper (Ifiemi et al, 2020).

The optimal network solution set is selected as the one with the least first index node pressure but not less than the stipulated minimum of 12.66mH<sub>2</sub>O. The head loss differentials relative to Node 1 for the cases of negligible minor losses and with the inclusion of the minor losses are shown in Table III.

TABLE III. HEAD LOSS DIFFERENTIAL RELATIVE TO NODE 1 (M)

Node	Case of Negligible Minor Losses	Case of Inclusion of Minor Losses
1	1.4988e-15 ≈ 0	2.77556e-15 ≈ 0
2	1.11279	1.98221
3	0.740244	1.31860
4	1.50818 (first index node)	2.68652 (first index node)
5	1.23838	2.20593
6	0.875311	1.55919
7	0.199845	0.355983
8	0.307516	0.547779
9	0.361755	0.644394
10	0.455124	0.810713
11	0.540061	0.962010

It was observed that the head loss differentials relative to Node 1, for all the nodes, increased for the case of inclusion of the minor losses over the differentials for the case of negligible minor losses. This was due to the additional head loss due to the pipe fittings in each pipe section for the former case.

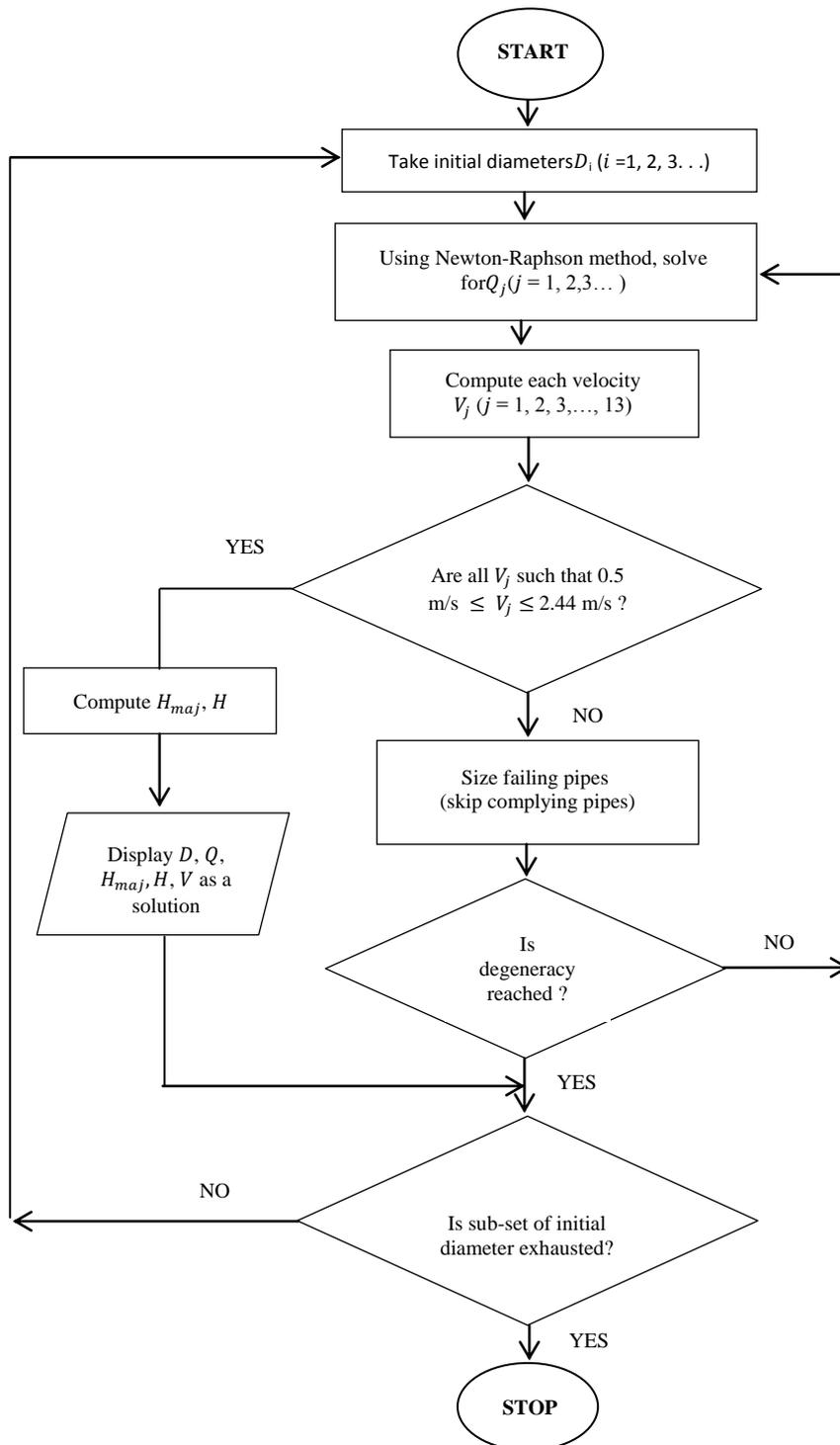


Figure 2. Pipe Size and Flow Determination Algorithm

## V. CONCLUSION

A water distribution network analysis was done, taking into account the minor losses in the system which had been neglected in an earlier study. The analysis was carried out using the Newton – Raphson multivariate method, facilitated by a MatLab code. Results show increases in the head loss differentials relative to the source node (Node 1), when compared with the results of the earlier study in which the minor losses were neglected. These increases require the storage tank elevation to be increased, in order to satisfy the distribution requirements.

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